THE OPEN UNIVERSITY
A SCIENCE FOUNDATION COURSE

UNIT 31 QUANTUM MECHANICS: ATOMS AND NUCLEI

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STUDY GUIDE

This Unit, which follows on directly from Unit 30, consists of only two components—the text and a television programme.

The text will probably take you longer than average to study (the text of Unit 30 is correspondingly shorter than average). If you find that you are short of time, we suggest that you drop Section 6 and, if necessary, Section 5 as well.

The Unit's TV programme 'Quantum leaps into the atom' specifically concerns material in Sections 3.1 and 4.3 of the text. The programme can be watched with profit at any stage in your studies of the Unit.

1 INTRODUCTION

As you saw way back in Units 11–12, the smallest portion of each chemical element is an atom of the element. Atoms are difficult to study because they are extremely tiny—typically 0.000 000 5 millimetre in diameter—but it is possible to learn a great deal about them from the electromagnetic radiation that they emit and absorb.

One might naively expect that atoms could emit and absorb radiation of a continuous range of wavelengths but, as you know, that turns out not to be true experimentally. In the second half of the nineteenth century, it was found that the emission spectrum and the absorption spectrum of each element include radiation of certain definite wavelengths (Figure 1). Moreover, these wavelengths uniquely characterize the element—they are the element's 'fingerprint'. For example, the wavelengths of the visible radiation emitted and absorbed by atomic hydrogen are approximately 410 nm, 434 nm, 486 nm and 656 nm (Figure 2). No other element can emit or absorb visible radiation of only these four wavelengths.

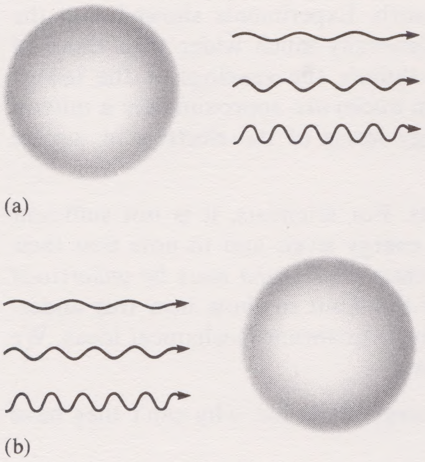

FIGURE 1 An electron in an atom can (a) emit and (b) absorb electromagnetic radiation of certain, definite wavelengths (and frequencies).

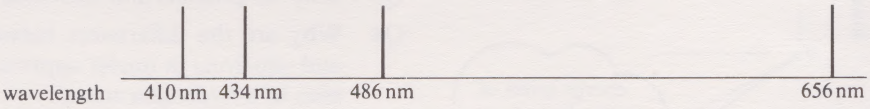

FIGURE 2 The visible emission spectrum of atomic hydrogen. (This spectrum is shown in the colour plate in Units 11–12.)

The Danish quantum physicist Niels Bohr was the first to interpret these absorption and emission spectra in terms of *energy levels.* In 1913, Bohr correctly suggested that the electrons in an atom of each element have characteristic energy levels (definite values of energy) and that, when an electron in an atom makes a transition to a lower energy level, radiation of a clearly defined wavelength is emitted (Figure 3a). Similarly, radiation of a definite wavelength is absorbed by an electron that makes a transition to a higher energy level (Figure 3b). Hence, *radiation of certain characteristic wavelengths can be emitted and absorbed by an atom because its electrons can make transitions ('quantum leaps') only between their energy levels. The spacings between these levels are, in turn, characteristic of the atom.*

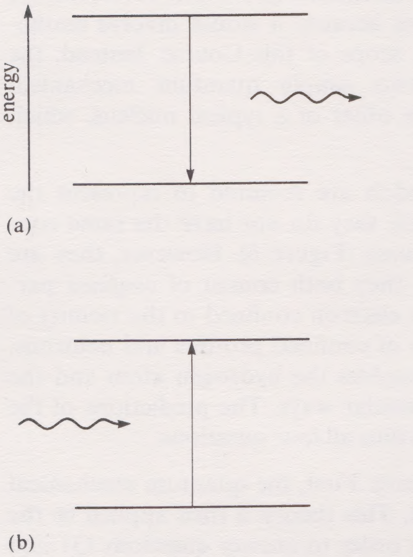

FIGURE 3 (a) When an electron in an atom makes a transition to a lower energy level, a photon is emitted with an energy equal to the spacing of the electron's energy levels; (b) an electron in an atom can absorb a photon with an energy equal to the spacing of two of the electron's energy levels, by making a transition from the lower of the two levels to the higher level.

From data on the absorption or emission spectrum of an element, it is possible to deduce the spacings of the energy levels of the electrons in an atom of the element. For example, the energy levels of the electron in a hydrogen atom are shown in Figure 4: notice that *the spacings of the lowest few levels are of the order of a few electronvolts* ($1\,\text{eV} \approx 1.602 \times 10^{-19}\,\text{J}$).

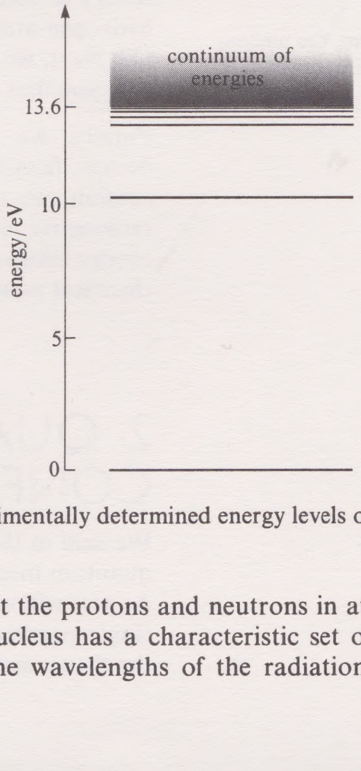

FIGURE 4 The experimentally determined energy levels of the electron in a hydrogen atom.

It was later found that the protons and neutrons in atomic *nuclei* also have energy levels: each nucleus has a characteristic set of energy levels whose spacings determine the wavelengths of the radiation that the constituent

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protons and neutrons can emit or absorb. Experiments showed that the spacings of nuclear energy levels are generally much wider than those of electronic energy levels. As a rule of thumb, the spacings of the lowest energy levels of protons and neutrons in nuclei are approximately a million times the spacings of the lowest energy levels of the electron in atomic hydrogen (Figure 5).

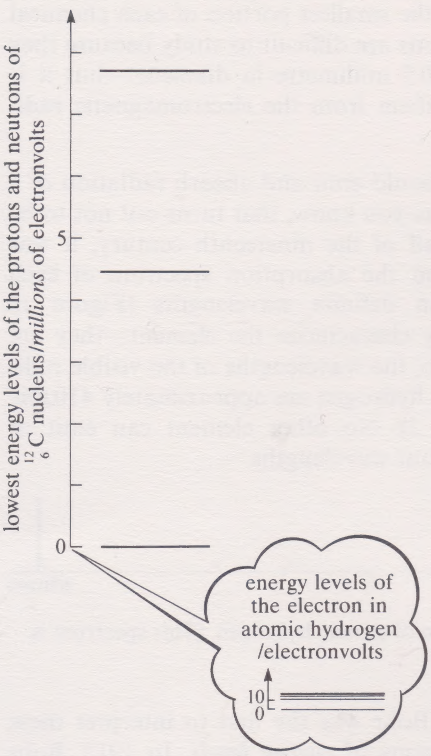


FIGURE 5 The spacings of the lowest energy levels of the protons and neutrons in a typical nucleus are generally very much wider than those of the electron in a hydrogen atom.

So much for the results of experiments. For scientists, it is not sufficient merely to observe atomic and nuclear energy levels and to note that their spacings are very different. *Somehow, these phenomena must be understood theoretically*. It is the main purpose of this Unit to show how this understanding can be derived in terms of simple quantum mechanical ideas. We shall seek to answer four basic questions:

Q1 Why do atomic electrons have energy levels (i.e. why can't they have *any* value of energy)?

Q2 Why are the differences between the lowest energy levels of the electron in a hydrogen atom a few electronvolts?

Q3 Why do protons and neutrons in atomic nuclei have energy levels?

Q4 Why are the differences between the lowest energy levels of protons and neutrons in nuclei approximately a million times those of the electron in a hydrogen atom?

In order to give rigorous answers to these questions, the behaviour of the atoms and nuclei should be investigated using the Schrödinger equation, one of the basic equations of quantum mechanics (Unit 30). However, we cannot pursue such an investigation here because it would involve mathematics that is considerably beyond the scope of this Course. Instead, the questions will be addressed using two simple quantum mechanical models—one of the hydrogen atom, the other of a typical nucleus, which consists of protons and neutrons.

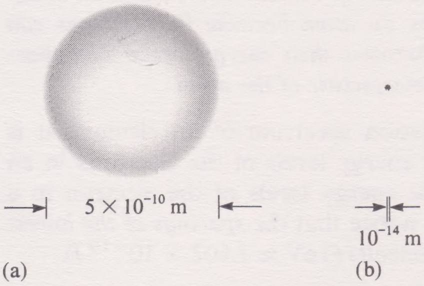


FIGURE 6 In each atom, the nucleus occupies only a tiny fraction of the atom's total volume.

You may think that very different models are required to represent the hydrogen atom and the nucleus: after all, they do not have the same constituents and they have very different sizes (Figure 6). However, they are similar in one very important respect—they both consist of *confined* particles. The hydrogen atom consists of an electron confined to the vicinity of a proton, and a typical nucleus consists of confined protons and neutrons. As you will see shortly, this similarity enables the hydrogen atom and the typical nucleus to be modelled in very similar ways. The predictions of the models will give extremely useful insights into all four questions.

The text of this Unit has a simple structure. First, the quantum mechanical theory of confined particles is developed. This theory is then applied to the hydrogen atom using a simple model, in order to answer questions Q1 and Q2. Next, we turn to nuclei and, after revising some basic facts about nuclei that you first met in Units 11–12, questions Q3 and Q4 are answered.

Finally, we shall discuss some topics in nuclear physics—radioactive decays, fission and fusion—using Einstein's equation $E = mc^2$. In this discussion, we shall explain in simple biological terms why the products of radioactive decays can be hazardous. Also, we shall comment on how energy released in nuclear reactions can be harnessed to provide a source of electrical power.

2 QUANTUM MECHANICS OF CONFINED PARTICLES

We said in the Introduction that we shall answer questions Q1 to Q4 using quantum mechanical models of the hydrogen atom and of a typical nucleus. As you will see in Sections 3 and 4, both models concern the behaviour of a single particle confined in three dimensions. The present Section provides the theoretical basis for the models.

First, we shall consider the comparatively simple case of a particle confined in only one dimension. Then, we shall move on to consider the analogous case of a particle confined in *three* dimensions. It is this three-dimensional case that will provide the basis for the atomic and nuclear models that will be developed later.

2.1 CONFINEMENT IN ONE DIMENSION

Let's begin by considering a particle undergoing the particular type of confinement in one dimension that you first met in the AV sequence in Unit 30. Remember, the particle is confined between two infinitely high, completely impenetrable, parallel plates (Figure 7). (From now on, we shall refer to this situation more briefly as 'one-dimensional confinement between parallel plates'.)

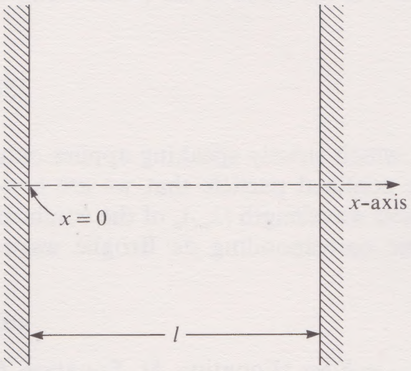


FIGURE 7 One-dimensional confinement between parallel plates.

We scarcely need to point out that the situation illustrated in Figure 7 is hypothetical. There is plainly no such thing as *infinitely* high plates, or as plates that are *completely* impenetrable. However, this artificiality is not important for our purposes: we are concerned only with the results that arise from considering the situation, because these results will be most useful later.

As you saw in Unit 30's AV sequence 'Wavefunctions of matter', one characteristic feature of the wavefunctions of a particle confined in one dimension between parallel plates is that their wavelengths are *quantized*, i.e. they can have only certain definite values. There is always a whole number of half-wavelengths between the wave's boundaries, and this condition can be expressed as an equation

$$\begin{pmatrix}\text{distance between}\\ \text{the boundaries of}\\ \text{the wavefunction}\end{pmatrix} = \text{whole number} \times \tfrac{1}{2} \begin{pmatrix}\text{wavelength}\\ \text{of the}\\ \text{wavefunction}\end{pmatrix} \quad (1)$$

The whole number (which could be 1 or 2 or 3 ... etc.) is called the **quantum number** of the wavefunction and it is denoted by n. Hence, if the wavelength of the wavefunction associated with quantum number n is $(\lambda_{\mathrm{wf}})_n$ and if the distance between the boundaries of the wavefunction is l (Figure 8), Equation 1 becomes

$$l = n \times \tfrac{1}{2}(\lambda_{\mathrm{wf}})_n \quad (2)$$

Equation 2 implies that

$$\boxed{(\lambda_{\mathrm{wf}})_n = \frac{2l}{n}, \text{ where } n = 1 \text{ or } 2 \text{ or } 3 \ldots \text{ etc.} \quad (3)}$$

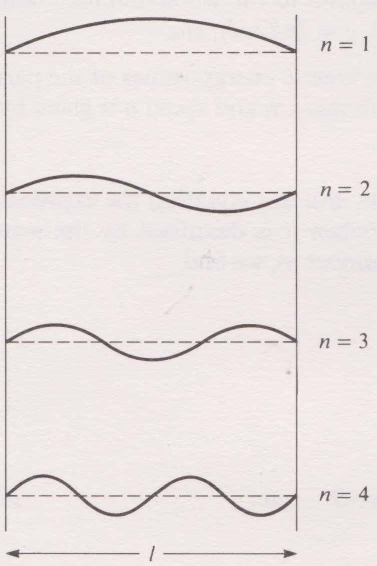


FIGURE 8 Four wavefunctions of a particle confined in one dimension between parallel plates (Figure 7).

Equation 3 is a simple expression for the quantized wavelengths $(\lambda_{\mathrm{wf}})_n$ of the wavefunctions of a particle confined in one dimension between parallel plates. The equation enables the allowed wavelengths to be calculated very easily; for example, if $l = 10^{-2}\,\mathrm{m}$, the two longest allowed wavelengths (which correspond to $n = 1$ and $n = 2$) of the wavefunctions are

$$\text{for } n = 1, \quad (\lambda_{\mathrm{wf}})_1 = \frac{2 \times 10^{-2}\,\mathrm{m}}{1} = 2 \times 10^{-2}\,\mathrm{m}$$

$$\text{for } n = 2, \quad (\lambda_{\mathrm{wf}})_2 = \frac{2 \times 10^{-2}\,\mathrm{m}}{2} = 10^{-2}\,\mathrm{m}$$

We shall now show that because the wavelengths of the wavefunctions are quantized (Equation 3), it follows that the total energy of the particle is also quantized, i.e. that the particle has energy levels. The link between the wavelengths of the wavefunctions and the particle's energy values can be made using de Broglie's formula.

QUANTIZATION

ENERGY LEVELS

STUDY COMMENT The following derivation of the particle's energy levels is important and you should work through it carefully. However, you are *not* required to commit it to memory.

In Unit 30, you saw that according to de Broglie's formula, the de Broglie wavelength λ_{dB} of a free particle with momentum of magnitude p is given by

$$\lambda_{\mathrm{dB}} = \frac{h}{p} \tag{4}$$

where h is Planck's constant, approximately 6.626×10^{-34} J s. Since p is defined by the expression $p = mv$, where m is the mass of the particle and v is its speed (Unit 3),

$$\lambda_{\mathrm{dB}} = \frac{h}{mv} \tag{5}$$

Now let us suppose that this equation, which strictly speaking applies *only* to free particles, can be applied to the confined particle that we are considering*. Also, suppose that each allowed wavelength $(\lambda_{\mathrm{wf}})_n$ of the confined particle's wavefunctions is equal to the corresponding de Broglie wavelength λ_{dB}:

$$(\lambda_{\mathrm{wf}})_n = \lambda_{\mathrm{dB}} \tag{6}$$

Since $(\lambda_{\mathrm{wf}})_n = 2l/n$ (Equation 3) and $\lambda_{\mathrm{dB}} = h/mv$ (Equation 5), Equation 6 implies that

$$\frac{2l}{n} = \frac{h}{mv} \tag{7}$$

where $n = 1$ or 2 or 3 ... etc. When Equation 7 is rearranged, we find

$$v = \frac{nh}{2ml} \tag{8}$$

Because h, m and l are constant for a given particle confined between a given pair of plates, Equation 8 predicts that the speed v of the particle is quantized. The allowed values of v correspond to the whole-number values of n: when $n = 1$, $v = h/(2ml)$; when $n = 2$, $v = 2h/(2ml)$; etc.

It is now quite straightforward to find the kinetic energy values of the particle. The kinetic energy E_{k} of a particle with mass m and speed v is given by

$$E_{\mathrm{k}} = \tfrac{1}{2}mv^2 \tag{9}$$

as you saw in Unit 9. If we now substitute into this equation the expression for the speed v of the confined particle (when it is described by the wavefunction characterized by the quantum number n), we find

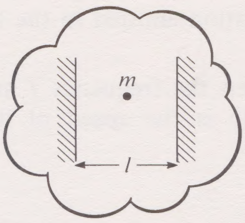
$$(E_{\mathrm{k}})_n = \tfrac{1}{2}m\left(\frac{nh}{2ml}\right)^2$$

i.e. $$(E_{\mathrm{k}})_n = \tfrac{1}{2}m\,\frac{n^2h^2}{4m^2l^2}$$

i.e. $$(E_{\mathrm{k}})_n = \frac{n^2h^2}{8ml^2}, \quad \text{where} \quad n = 1 \text{ or } 2 \text{ or } 3 \ldots \text{ etc.} \tag{10}$$

---

* You may think that the de Broglie formula cannot be applied to the confined particle because such an application would imply that the momentum of the particle were known with perfect accuracy, in contradiction with the uncertainty principle ($\Delta x \Delta p_x \geqslant h/(4\pi)$, Unit 30). However, this objection is not valid: it is assumed in the derivation that the *magnitude* p of the particle's momentum is known precisely, but the *direction* in which the particle is travelling has not been specified. Hence, there *is* an uncertainty in the x-component of the particle's x-component of momentum. This uncertainty Δp_x has the value p: the x-component of momentum would be $p_x = +p$ if the particle is travelling in one direction and $p_x = -p$ if it is travelling in the opposite direction, so the 'spread' of p_x is $p - (-p) = 2p$, and so $\Delta p_x = p$ (half the spread).



The particle is free to move between the plates so its energy depends only on its speed, not on its position. The energy that the particle has because of its position—its *potential* energy—is therefore a constant. The value of this constant may be taken to be zero (remember from Unit 9 that only *relative* values of energy are important, not absolute values). Hence, the *total* energy E of the particle (when it is described by the wavefunction characterized by the quantum number n) is given by the right-hand side of Equation 10

$$\boxed{E_n = \frac{n^2h^2}{8ml^2}, \quad \text{where} \quad n = 1 \text{ or } 2 \text{ or } 3 \ldots \text{etc.} \qquad (11)}$$

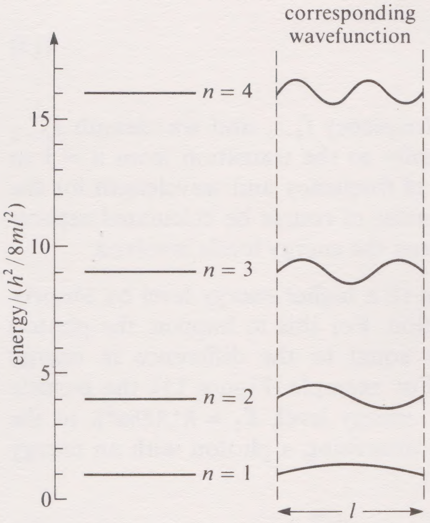


FIGURE 9 The four lowest energy levels and the corresponding wavefunctions of a particle confined in one dimension between parallel plates (Figure 7).

This is the result we have been seeking: it gives the total energy values of a particle confined in one dimension between parallel plates. Because the quantum number n is a whole number, it follows that the quantity n^2 in Equation 11 is also a whole number. Each value of n therefore corresponds to a certain, definite value of total energy E: when $n = 1$, $E_1 = h^2/(8ml^2)$; when $n = 2$, $E_2 = 4h^2/(8ml^2)$ and so on. The energy is therefore **quantized**—the particle has **energy levels**.

The underlying reason for the quantization of the particle's energy is that a *whole* number of half-wavelengths of the particle's wavefunctions have to 'fit' between the confining plates (Equation 1). It is because the particle is *confined* that its energy is quantized.

The lowest four energy levels of a particle confined in one dimension are shown in Figure 9, together with its corresponding wavefunctions. Remember that these energy levels and wavefunctions apply *only* to a particle that can move in only one dimension between parallel plates.

It is worthwhile to pause briefly to examine further the energy levels of the particle. First, consider its lowest possible energy. According to commonsense ideas, it should be expected that the minimum energy of the particle should be zero—this would be the energy of the particle when it is permanently stationary between the plates. However, according to Figure 9 the particle cannot have zero total energy—the particle's lowest possible energy is $h^2/(8ml^2)$. This is the total energy of the particle when the minimum number (i.e. one) of half-wavelengths of its wavefunctions 'fits' between the plates.

Now consider the transitions that the particle can make between its energy levels. If the particle makes a transition to a lower energy level, its energy decreases and this energy is 'carried off' by a single emitted photon. For example, if the particle makes a transition from the $n = 3$ energy level, $E_3 = 9h^2/(8ml^2)$, to the $n = 2$ energy level, $E_2 = 4h^2/(8ml^2)$, it will emit a single photon with an energy given by the difference between the two energies, which we can label $\Delta E_{3\to 2}$:

$$\Delta E_{3\to 2} = \frac{9h^2}{8ml^2} - \frac{4h^2}{8ml^2}$$

$$\text{i.e.} \quad \Delta E_{3\to 2} = \frac{5h^2}{8ml^2} \qquad (12)$$

(Figure 10). The radiation emitted of course has a characteristic frequency and wavelength.

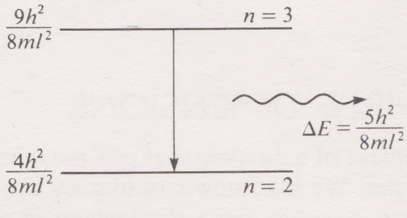


FIGURE 10 When a particle makes a transition from the energy level $9h^2/(8ml^2)$ to the lower energy level $4h^2/(8ml^2)$, a single photon with energy $5h^2/(8ml^2)$ is emitted.

☐ What is the *frequency* $f_{3\to 2}$ of the radiation emitted in the transition from $n = 3$ to $n = 2$ shown in Figure 10?

■ The standard formula $E = hf$ for the energy of a photon, implies in this case that $\Delta E_{3\to 2} = hf_{3\to 2}$, so according to Equation 12

$$\frac{5h^2}{8ml^2} = hf_{3\to 2}$$

$$\text{i.e.} \quad f_{3\to 2} = \frac{5h}{8ml^2} \qquad (13)$$

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☐ What is the *wavelength* $\lambda_{3\to2}$ of the radiation emitted in the transition shown in Figure 10?

■ The standard relationship $c = f\lambda$ between the frequency f and wavelength λ of electromagnetic radiation (c is the speed of light in a vacuum) implies in this case that

$$\lambda_{3\to2} = \frac{c}{f_{3\to2}} \tag{14}$$

Hence, Equations 13 and 14 imply that

$$\lambda_{3\to2} = \frac{8ml^2c}{5h} \tag{15}$$

Remember that these expressions for frequency $f_{3\to2}$ and wavelength $\lambda_{3\to2}$ of the emitted radiation apply specifically to the transition from $n = 3$ to $n = 2$ shown in Figure 10. The values of frequency and wavelength for the radiation emitted in other transitions must of course be calculated separately, by considering the difference between the energy levels involved.

The particle can also make a transition to a higher energy level by absorbing a photon of electromagnetic radiation. For this to happen, the photon must have an energy that is exactly equal to the difference in energy between the energy levels concerned. For example (Figure 11), the particle can make a transition from the $n = 1$ energy level, $E_1 = h^2/(8ml^2)$, to the $n = 3$ energy level, $E_3 = 9h^2/(8ml^2)$, by absorbing a photon with an energy $\Delta E_{1\to3}$ where

$$\Delta E_{1\to3} = \frac{9h^2}{8ml^2} - \frac{h^2}{8ml^2} \quad \text{i.e.} \quad \Delta E_{1\to3} = \frac{8h^2}{8ml^2}$$

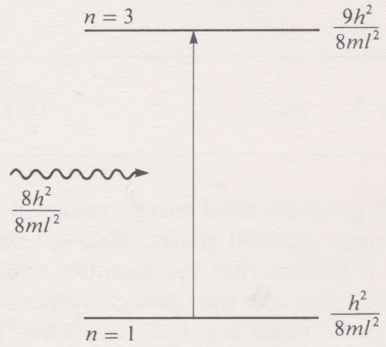


FIGURE 11 When a particle that occupies the energy level $h^2/(8ml^2)$ absorbs a photon with energy $8h^2/(8ml^2)$, the particle makes a transition to the higher energy level $9h^2/(8ml^2)$.

Now, before you move on to Section 2.2, which concerns confinement in *three* dimensions, have a go at ITQs 1 and 2 in order to check that you have understood the important points covered so far.

ITQ 1 Explain in a few sentences (*without* using equations) why, according to quantum mechanics, a particle confined in one dimension between parallel plates has energy levels.

ITQ 2 A particle that has a mass of 1.25×10^{-27} kg is confined in one dimension between parallel plates that are separated by 6.63×10^{-5} m.

(a) Using Equation 11, show that the energy levels of the particle are given by

$$E_n = (10^{-32}\,\text{J}) \times n^2$$

where n is a whole number ($n = 1$ or 2 or 3 ... etc.)

(b) Show on Figure 12 the $n = 1$ and $n = 3$ energy levels of the particle (the $n = 2$ energy level is already shown).

(c) What is the energy of the photon of electromagnetic radiation that is emitted when the particle makes a transition from the $n = 3$ energy level to the $n = 1$ energy level?

(d) What are the frequency and wavelength of the radiation emitted in the transition specified in part (c)?

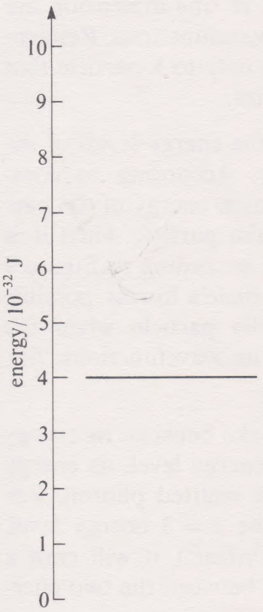


FIGURE 12 See ITQ 2.

2.2 CONFINEMENT IN THREE DIMENSIONS

In Section 2.1, we discussed the confinement of a particle that can move in only one dimension between parallel plates. We shall now consider an analogous situation in which a particle can move in *three* dimensions in a hollow cube (Figure 13) whose sides (of length L) are completely impenetrable, so that the particle can never escape. Because the motion of the particle is three-dimensional, it follows by definition that the position of the particle can be specified with respect to *three* axes (x, y and z in Figure 13).

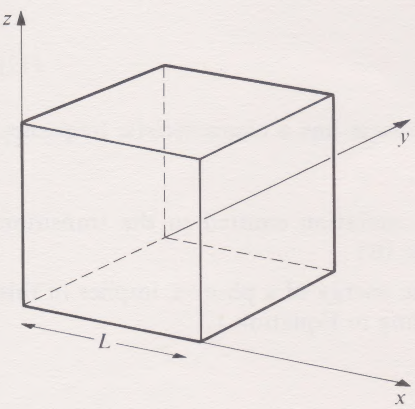


FIGURE 13 A hollow cube whose sides each have length L and whose faces are impenetrable.

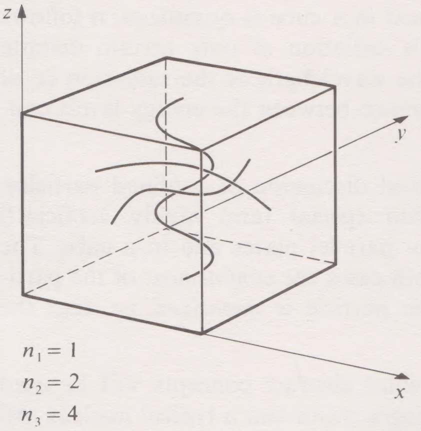


FIGURE 14 For a particle confined in a hollow cube (Figure 13), a whole number of half-wavelengths of the particle's wavefunction 'fit' between each pair of the cube's faces. In the particular case of the wavefunction shown schematically here, one half-wavelength 'fits' between one pair of faces, two half-wavelengths 'fit' between another pair, and four half-wavelengths 'fit' between the remaining pair.

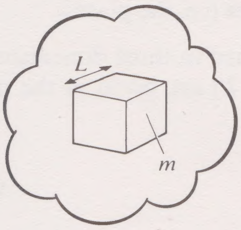
According to quantum mechanics, the behaviour of any particle is determined by its wavefunction. For a particle that can move in three dimensions, its possible wavefunctions are very difficult to visualize properly. However, the quantum mechanical principles of confinement in three dimensions are practically identical with those that underlie the theory of one-dimensional confinement. You saw in Section 2.1 that, in the case of one-dimensional motion, a whole number of half-wavelengths of the particle's wavefunction must 'fit' between the plates (for a given wavefunction, this whole number is given by its quantum number n). This quantization of particle's associated wavelength led directly to the quantization of the particle's energy.

In the case of the three-dimensional motion of a particle in a cube, a whole number of half-wavelengths of the particle's wavefunctions must 'fit' between *each* of three pairs of opposite faces of the cube (an example is illustrated schematically in Figure 14). Hence, the wavefunction is characterized by three quantum numbers n_1, n_2 and n_3, which give respectively the number of half-wavelengths of the wavefunctions that fit in the x-, y- and z- directions (Figure 14). Each of these quantum numbers can equal any whole number.

The quantization of the wavelengths of the particle leads to the quantization of its energy. It can be shown that the energy of any particle of mass m confined in a cube with sides of length L is given by

$$E = \frac{n_1{}^2h^2}{8mL^2} + \frac{n_2{}^2h^2}{8mL^2} + \frac{n_3{}^2h^2}{8mL^2} \tag{16}$$

It would have been more logical (and more consistent with the notation used in Equation 11) if we had denoted this energy by E_{n_1,n_2,n_3}, but that notation would have been just *too* cumbersome!



Because n_1, n_2 and n_3 in Equation 16 are whole numbers, it follows that the total energy E of the particle can have only certain, definite values: the particle's energy is quantized. Notice the similarity between Equation 16 (which concerns *three*-dimensional confinement) and Equation 11 ($E_n = n^2h^2/8ml^2$), the corresponding expression for *one*-dimensional confinement.

It is convenient to re-express Equation 16, by taking out the common factor $h^2/(8mL^2)$ from each of the three terms in the equation:

$$\boxed{E = \frac{h^2}{8mL^2}(n_1{}^2 + n_2{}^2 + n_3{}^2) \quad \begin{array}{l} n_1 = 1 \text{ or } 2 \text{ or } 3 \ldots \text{etc.} \\ n_2 = 1 \text{ or } 2 \text{ or } 3 \ldots \text{etc.} \\ n_3 = 1 \text{ or } 2 \text{ or } 3 \ldots \text{etc.} \end{array}} \tag{17}$$

This equation (identical in content with Equation 16, but more tidily expressed) is extremely important and it will be used frequently in this Unit. However, there is no need for you to memorize it.

Figure 15 shows the lowest few energy levels of the particle confined in a cube. As you can see from the Figure, some energy levels of the particle in the cube correspond to more than one set of quantum numbers (n_1, n_2, n_3). For example, the energy level $E = 6h^2/(8mL^2)$ corresponds to three different wavefunctions: the ones specified by ($n_1 = 1$, $n_2 = 2$, $n_3 = 1$), ($n_1 = 2$, $n_2 = 1$, $n_3 = 1$) and ($n_1 = 1$, $n_2 = 1$, $n_3 = 2$). This phenomenon, in which an energy level corresponds to more than one wavefunction, is called **degeneracy**. The wavefunctions that correspond to the energy level are said to be *degenerate*.

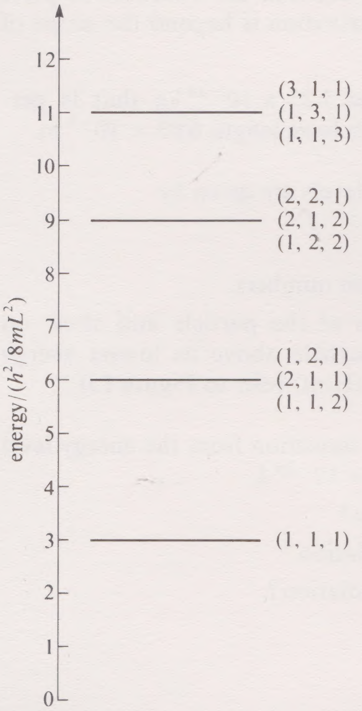


FIGURE 15 The four lowest energy levels of a particle of mass m confined in a cube whose sides each have length L.

ITQ 3 (a) What is the energy of a particle of mass m in a cube with sides of length L when the particle's wavefunction is characterized by the quantum numbers $n_1 = 2$, $n_2 = 2$, $n_3 = 2$? Show this energy level on Figure 15.

(b) Is the energy level $E = 9h^2/(8mL^2)$ shown in Figure 15 degenerate? Explain your answer briefly.

Because the energy of a particle confined in a cube is quantized, it follows that the particle can emit and absorb radiation of only certain definite wavelengths. For a given transition, the wavelength of the radiation is, of course, determined by the energy difference between the energy levels concerned.

That concludes this quantum mechanical discussion of confined particles. We have considered theoretically two special (and highly artificial!) situations—a particle confined between parallel plates and in a cube. The crucial point to remember is that in both cases the *confinement* of the particle has implied that the energy of the particle is *quantized*, i.e. that the particle has energy levels.

In the next two Sections, these somewhat abstract concepts will be used when we formulate models of the hydrogen atom and a typical nucleus. All the abstract theoretical work you have done in this Section will be handsomely rewarded shortly when we use our models to account for data from experiments.

SUMMARY OF SECTION 2

1 The energy levels of a particle of mass m confined in one dimension between infinite, parallel, impenetrable plates that are separated by distance l are given in the usual notation by

$$E_n = \frac{n^2h^2}{8ml^2} \qquad (11)^*$$

This expression is derived by 'fitting' a whole number of half-wavelengths of the particle's wavefunctions between the boundaries (i.e. the plates).

2 The energy levels of a particle of mass m confined in three dimensions in a hollow cube (with impenetrable sides of length L) are given in the usual notation by

$$E = \frac{h^2}{8mL^2}({n_1}^2 + {n_2}^2 + {n_3}^2) \qquad (17)^*$$

This expression can be derived by 'fitting' a whole number of half-wavelengths of the particle's wavefunctions between the cube's three pairs of faces (e.g. Figure 14). However, such a derivation is beyond the scope of this Course.

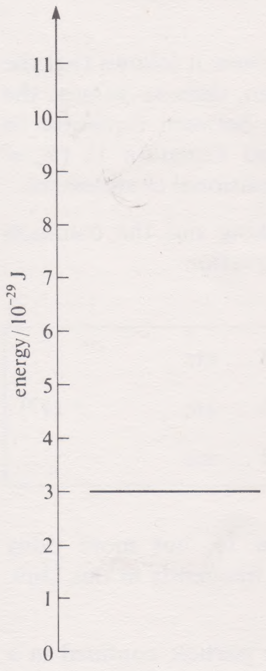


FIGURE 16 See SAQs 1 and 2.

SAQs 1 and 2 concern a particle of mass 1.25×10^{-28} kg that is permanently confined in a cube whose sides each have length 6.63×10^{-6} m.

SAQ 1 (a) Show that the particle's energy levels are given by

$$E = 10^{-29}\,\text{J} \times ({n_1}^2 + {n_2}^2 + {n_3}^2)$$

where n_1, n_2 and n_3 are the particle's quantum numbers.

(b) Calculate the four lowest energy levels of the particle and show on Figure 16 the three energy levels of the particle above its lowest energy level, which is already shown on the Figure. (Hint: refer to Figure 15).

SAQ 2 Suppose that the particle makes a transition from the energy level $E = 11 \times 10^{-29}$ J to the energy level $E = 6 \times 10^{-29}$ J.

(a) What is the energy of the emitted photon?

(b) What is the frequency of the emitted radiation?

(c) What is the wavelength of the emitted radiation?

3 UNDERSTANDING ATOMIC ENERGY LEVELS

In this Section, the theoretical ideas introduced in the previous Section will be used to derive insights into atomic energy levels. First, the atom with the simplest possible structure—the hydrogen atom—will be considered in detail. A model of this atom will be formulated and the predictions of the model will be compared with the corresponding data from experiments. Finally, we shall look at more advanced atomic models.

3.1 A SIMPLE MODEL OF THE HYDROGEN ATOM (TV programme)

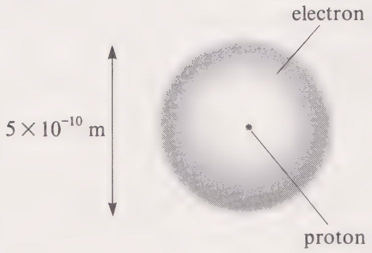


FIGURE 17 A visualization of a hydrogen atom.

A hydrogen atom has a diameter of approximately 5×10^{-10} m and it consists of an electron moving around a proton (Figure 17). The charge of the electron, approximately -1.602×10^{-19} C, is exactly opposite to that of the proton so the atom has no net electrical charge. Coulomb's law of electrostatics (Unit 9) tells us that the magnitude of the attractive electrostatic force $F_{\rm el}$ between the electron and the proton varies with their separation r

$$F_{\rm el} \propto \frac{1}{r^2} \tag{18}$$

The separation r of the two particles varies continuously, so the electrostatic force between the electron and the proton also changes continuously. There is also an attractive gravitational force between the two particles but its magnitude is negligible compared with that of the electrostatic force.

Experiments show that hydrogen atoms emit and absorb radiation of certain, definite wavelengths and this tells us that the energy of the atom is *quantized*. How can this energy quantization be understood theoretically?

It would be difficult to understand this phenomenon using a *complete* description of the atom, because such a description would have to take into account not only the masses, charges and spins of its constituent particles, but also the continuously changing electrostatic force between the electron and the proton. Such an investigation will not be attempted here as it would be beyond the scope of this Course. Instead, the energy of the electron in a hydrogen atom will be studied using a *model* of the atom.

Before the model is described in detail, it is worthwhile to pause to review the meaning of the term 'model'. In Unit 1, it was defined to be 'an artificial construction invented to represent or to simulate the properties, the behaviour, or the relationship between the individual parts of the real entity being studied'. The 'real entity' that is being studied here is the hydrogen atom, which we shall represent by the model shown in Figure 18. The atom is modelled as a hollow cubic box (with sides of 5.00×10^{-10} m, approximately the diameter of the atom) containing a particle with the same mass as the electron (approximately 9.11×10^{-31} kg). By representing the hydrogen atom in this way, only two of its characteristics are being taken into account:

- its size;
- the mass of its constituent electron.

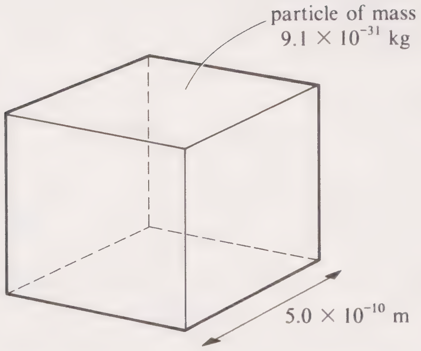


FIGURE 18 A simple model of the hydrogen atom.

You have probably already guessed why we have chosen this particular model: in Section 2, the confinement of a particle in a cubic box has already been discussed. Hence, in choosing this model, we can use results that have already been derived. The model is chosen for its simplicity and convenience.

It is important to bear in mind that, in the formulation of the model, some simplifying assumptions have been made. For example, the physical presence of the proton in the atom is ignored and it is assumed that the atom has the shape of a cube. The way in which the magnitude of the attractive electrostatic force between the proton and the electron depends on their separation has not been taken into account. Also, the spin of the electron has been ignored. (Experiments also show that the proton has spin, but that is also disregarded.)

You may well be thinking that some or all of these assumptions are unreasonable. It is indeed only prudent to be wary of the validity of each assumption in this (or any other) model, but it is also wise to defer judgement on the model until its predictions have been compared with the corresponding experimental results. Only after the comparison has been made is it reasonable to assess the model's worth. If its predictions turn out to be in flagrant conflict with data, then the model must be dropped and another should be formulated. Alternatively, if the predictions are found to be in good agreement with experiment, it would be reasonable to make the modifications necessary to improve the agreement.

ITQ 4 What would you consider to be *reasonable* agreement between the predictions of the model for the energy values of the atom and the experimentally determined values? For example, do you think it reasonable to expect the two to agree exactly?

(This is really a personal question—only you can say what you consider to be a reasonable outcome of the comparison. However, there are some important points to bear in mind when answering an open-ended question like this: don't forget to look at our answer before you move on!)

Now, at last, it is time to apply the model. We shall compare the energy values predicted by the model of the hydrogen atom with the energy levels of atomic hydrogen that are observed experimentally.

First, consider the model (Figure 18): the hydrogen atom is represented by a particle of mass 9.11×10^{-31} kg confined in a hollow cube, whose sides each are of length 5.00×10^{-10} m. Note that these input data are given to three significant figures; at the end of the calculation we shall 'round down' the predictions to *two* significant figures (remember the last significant figure is always unreliable).

You know from Section 2 that a particle of mass m in a cube whose sides have length L has *energy levels* given by

$$E = \frac{h^2}{8mL^2}(n_1{}^2 + n_2{}^2 + n_3{}^2) \qquad (17)^*$$

where n_1, n_2, n_3 are each separately equal to a whole number. Here, immediately, is a success of the model: it predicts that the energy of the system it represents—the hydrogen atom—is quantized, in agreement with experimental observations. The particle in the cube has energy levels because it is confined, so it is reasonable to infer that the *electron in a hydrogen atom has energy levels because it is confined.*

Let us now examine the *numerical* predictions of the model. Because Planck's constant $h \approx 6.63 \times 10^{-34}$ J s and because $m = 9.11 \times 10^{-31}$ kg and $L = 5.00 \times 10^{-10}$ m in the model, Equation 17 says that the energy levels of the particle in the box are given by

$$E = \frac{(6.63 \times 10^{-34}\,\mathrm{J\,s})^2}{8 \times (9.11 \times 10^{-31}\,\mathrm{kg}) \times (5.00 \times 10^{-10}\,\mathrm{m})^2}(n_1{}^2 + n_2{}^2 + n_3{}^2) \qquad (19)$$

It is straightforward to show that

$$E = (2.41 \times 10^{-19}\,\mathrm{J}) \times (n_1{}^2 + n_2{}^2 + n_3{}^2) \qquad (20)$$

(You should check with your calculator that Equation 20 follows from Equation 19.) As you saw in Unit 9, it is convenient in atomic science to measure energy in units of electronvolts (eV) rather than in the standard units of joules (J):

$$1\,\mathrm{eV} \approx 1.602 \times 10^{-19}\,\mathrm{J} \qquad (21)$$

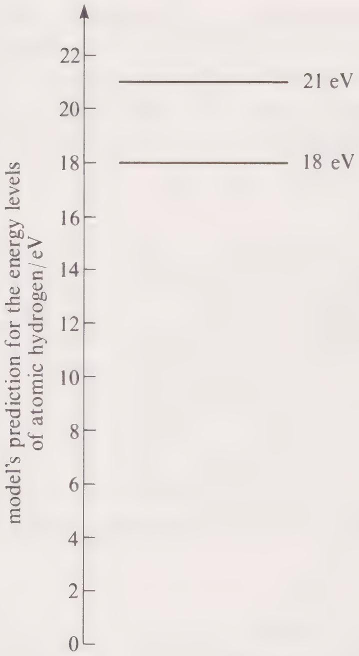

model's prediction for the energy levels of atomic hydrogen/eV
22
20
18
16
14
12
10
8
6
4
2
0
21 eV
18 eV

FIGURE 19 Prediction of the simple model of the hydrogen atom (Figure 18) for the six lowest energy levels of atomic hydrogen. In ITQ 5, you are asked to draw in the four lowest energy levels.

Hence, using Equation 21, Equation 20 can be re-expressed, to two significant figures, as

$$\boxed{E = 1.5\,\text{eV} \times (n_1^2 + n_2^2 + n_3^2) \qquad (22)}$$

Equation 22 gives the energy levels of the particle in the box—*the equation is the model's prediction for the energy levels of the electron in a hydrogen atom.*

ITQ 5 Calculate the four lowest energy levels of the particle and mark them on Figure 19 (the two energy levels above the lowest four are already marked on the Figure). (Hint: it may help you to look back to Figure 15).

The spacings of the four lowest energy levels of the electron in the hydrogen atom predicted by the model (Figure 19) are compared in Figure 20 with the experimentally determined spacings. Notice that in Figure 20 the lowest energy levels are aligned. This alignment can be made because the *numerical* values associated with the levels are not important, because *absolute* energy values cannot be measured; only their *differences* can be measured (e.g. from atomic hydrogen's emission spectrum and absorption spectrum). Thus, as you saw in Unit 9, only the *relative* values of energies are important.

As you can see from Figure 20, the agreement between the model and experiment is remarkably good—the spacings of the lowest levels agree to well within an order of magnitude. For higher energy levels, the comparison is not so successful: the model predicts that there are an infinite number of energy levels that correspond to the infinite number of possible values of $n_1^2 + n_2^2 + n_3^2$ in Equation 22 (remember n_1, n_2 and n_3 can each equal any positive number). However, experiment shows that the electron in a hydrogen atom has a continuum (i.e. a *continuous*, non-quantized range) of energies, starting approximately 13.6 eV above the lowest energy level. The model cannot account for this because when the electron has an energy in the continuum of energies, it is *not* confined. Such a situation cannot be described within the scope of the model, according to which the electron is always confined.

The prediction for the spacings of the lowest few energy levels was successful because the model took into account the three factors that are most important in determining the spacings of the energy levels—the size of the atom, the mass of the confined electron and the value of Planck's constant. This answers question Q2 in the Introduction: *the differences between the lowest few energy levels of the electron in a hydrogen atom (roughly a few electronvolts) are determined mainly by the size of the atom, by the mass of the electron and by the value of Planck's constant.*

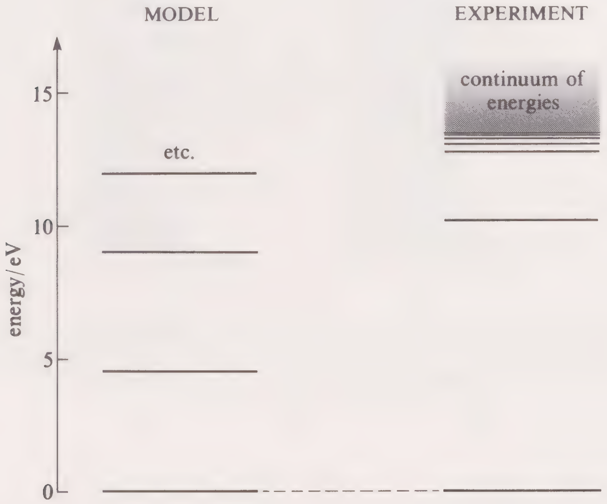


FIGURE 20 Comparison between the model's prediction for the four lowest energy levels of atomic hydrogen and the experimentally determined energy levels of atomic hydrogen. The lowest energy levels have been aligned—this can be done because only *relative* energies (not their actual numerical values) are important. In other words, the position of the zero of energy is arbitrary (Unit 9). Notice that in this Figure the lowest energy level in the model has been aligned with the corresponding experimental value by subtracting 4.5 eV from each of the energy values in Figure 19.

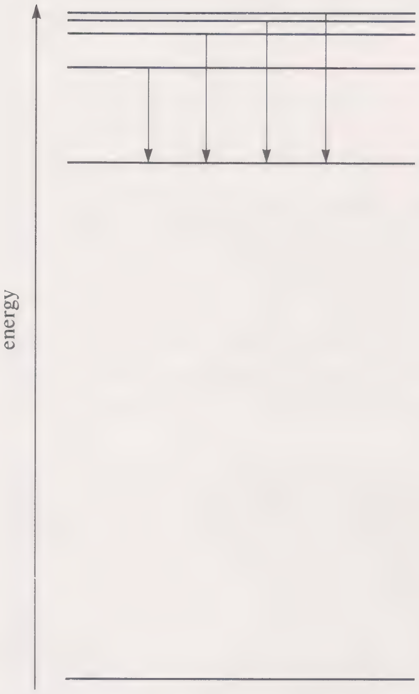


FIGURE 21 The transitions that are found experimentally to be responsible for the four *visible* spectral lines of atomic hydrogen.

It is a useful exercise to compare the model's numerical predictions for the energies of the photons emitted from atomic hydrogen with the corresponding experimental data. We now want you to carry out this exercise in order that you can see how well the model accounts for the visible part of the spectrum of atomic hydrogen, which is studied in part of the physics experiment at Summer School (and in the Fraunhofer spectrum experiment in Units 11–12).

The visible spectrum of atomic hydrogen consists of four spectral lines that have wavelengths of approximately 656 nm, 486 nm, 434 nm and 410 nm, and these respectively correspond to photon energies of 1.9 eV, 2.6 eV, 2.9 eV and 3.0 eV. The transitions that give rise to these lines are shown in Figure 21: they correspond to transitions to the second lowest energy level from the third, fourth, fifth and sixth lowest energy levels. In ITQ 6, you will see how well the measured values of the energies of the photons emitted in these transitions agree with the corresponding predictions of the model. It will be assumed that the corresponding transitions in the model are also to the second lowest energy level from the four levels immediately higher in energy (Table 1).

ITQ 6 (a) Use Figure 19 to find the energies of the photons emitted in the transitions of the particle in the atom-sized box to the second lowest energy level from the third, fourth, fifth and sixth lowest energy levels. Enter your results in the spaces provided in Table 1.

(b) Do you think that the agreement between the predictions of the model and the corresponding measurements is good enough for the model to be regarded as useful?

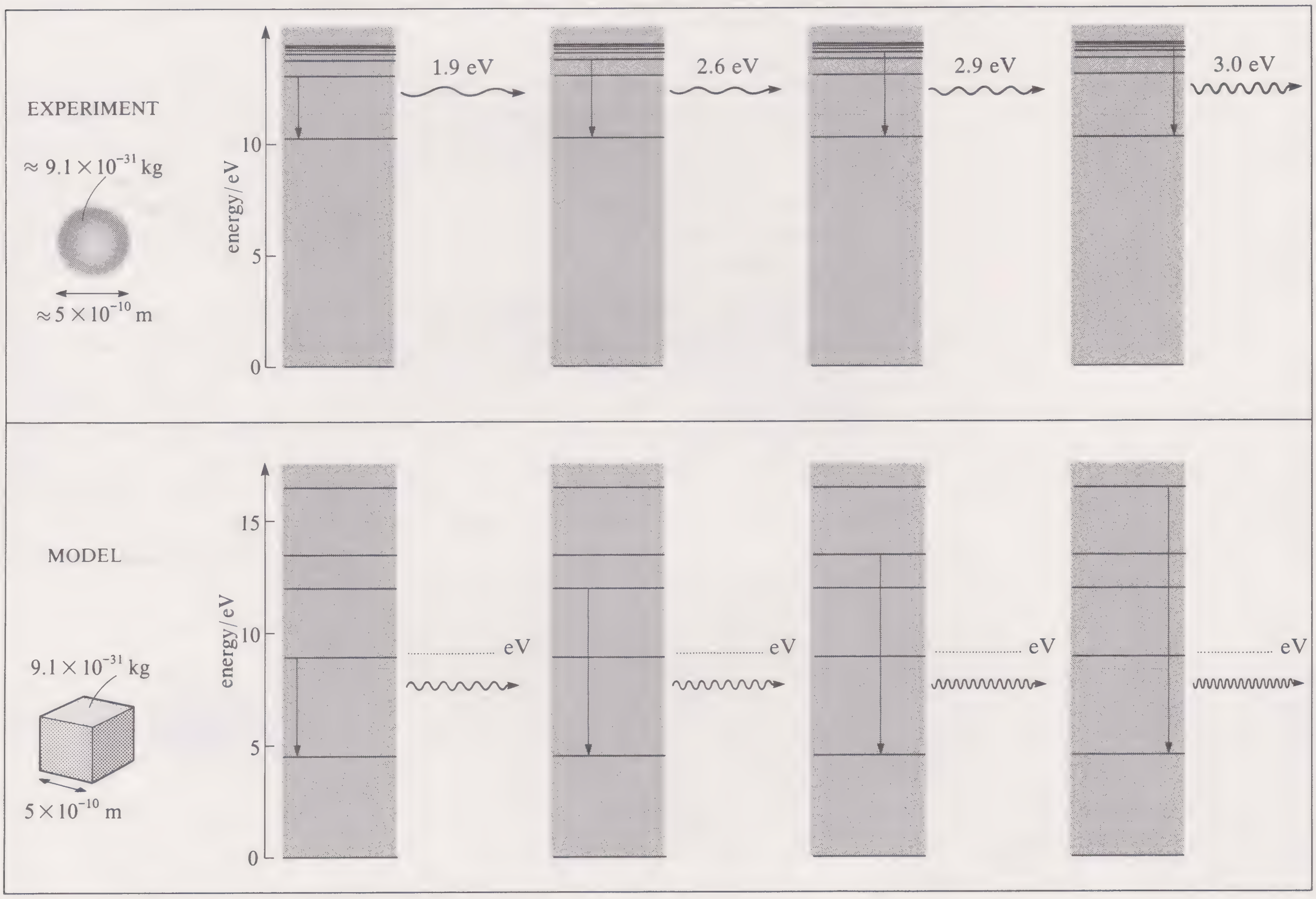


TABLE 1 The transitions associated with the visible emission spectrum of atomic hydrogen, compared with the corresponding transitions between energy levels predicted by the particle-in-a-box model of the hydrogen atom. The comparison between the Experiment and Model is shown in this Table, where once again 4.5 eV has been subtracted from each of the energy values in the Model.

3.2 MORE ADVANCED ATOMIC MODELS

It would be wrong to leave you with the impression that the most advanced theoretical descriptions of atoms involve modelling them as cubes—such an impression would be very misleading. Modern theoretical treatments are very much more subtle and complicated than our particle-in-a-box model.

It is possible to give a very good account of energy levels of the electron in a hydrogen atom (and of the corresponding wavefunctions) using the Schrödinger equation (Unit 30). According to this equation, the wavefunction of an electron in a hydrogen atom can be characterized by three quantum numbers—the principal quantum number n, the second quantum number l and the orbital magnetic quantum number m_l*. *Note that these quantum numbers have nothing to do with the quantum numbers* n_1, n_2, n_3 *of a particle confined in a cubic box.*

You met the quantum numbers n, l and m_l in Units 11–12, where you saw that each set of quantum numbers corresponds to a particular electron orbital, which gives the probability of detecting in each region the electron when it has a given set of quantum numbers (e.g. Figure 22). Each orbital can be calculated using the Schrödinger equation.

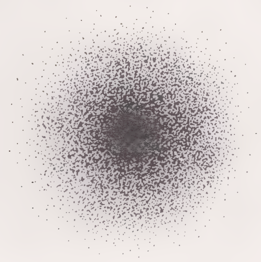


FIGURE 22 The orbital of the electron in a hydrogen atom when the electron has the lowest possible energy.

So much for the energy levels of the electron in a hydrogen atom; what about the energies of electrons in heavy atoms (i.e. atoms that contain more than one electron)? In common with the hydrogen atom, heavy atoms have energy levels—quantized values of energy—and this quantization can be understood intuitively by considering the atoms' constituent electrons. Because these electrons are *confined*, they have energy levels. The differences between the energy levels of heavy atoms cannot be predicted as easily as those of atomic hydrogen, because in a heavy atom there are complex mutual interactions of the electrons moving around the nucleus. (No such complexities occur in the hydrogen atom, of course, because it contains only *one* electron.) The mutual interactions can be taken into account only by using computers to apply the Schrödinger equation to the electrons. When this is done it is possible to predict energy levels and wavefunctions with high accuracy.

The results of calculations of this kind show that the orbitals of heavy atoms broadly resemble those of hydrogen, but there are important differences between the energies of electrons in hydrogen and in heavy atoms. First, the higher charge of the nuclei of heavy atoms ensures that the inner electrons are held more tightly than is the electron in a hydrogen atom. Second, the energy levels of electrons with the same principal quantum number n are no longer degenerate as they are in hydrogen (Units 11–12) but depend on the second quantum number l. Electrons with the same values of the principal quantum number n and the second quantum number l, but different values of m_l, have approximately the same energies. The

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* The spin magnetic quantum number, and indeed the actual phenomenon of electron spin, can be understood only by using quantum mechanics in conjunction with the special theory of relativity.

NUCLEUS

MASS NUMBER

ATOMIC NUMBER

result is that the energy level diagram for a heavy atom, such as the sodium atom, contains more levels of *different* energies than that of hydrogen, and the spacings between the lowest levels are greater than the corresponding spacings in the energy level diagram for hydrogen (Figure 23).

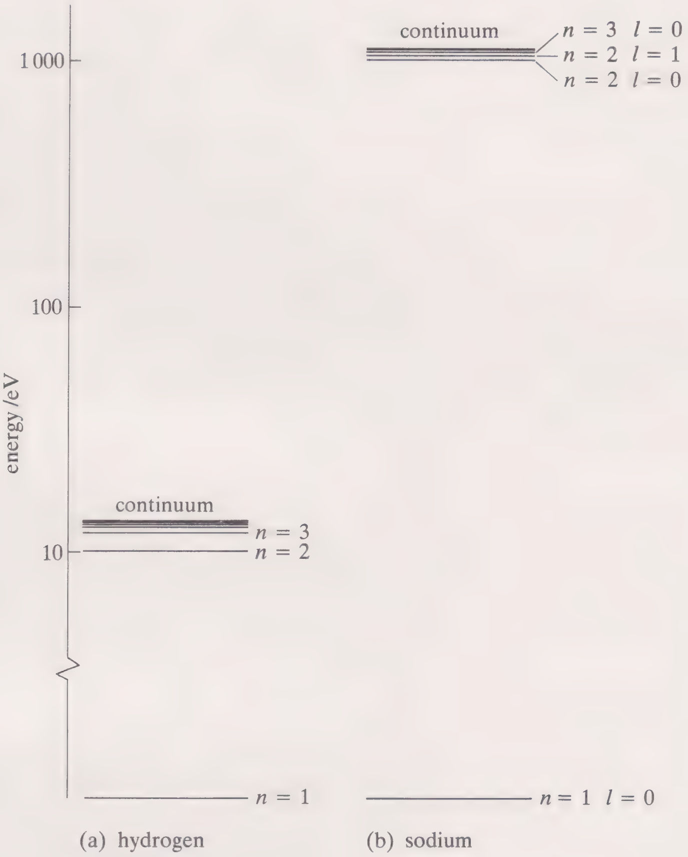


FIGURE 23 Electron energy-level diagrams for (a) hydrogen and (b) sodium. Note that the energy scale is logarithmic, to accommodate the large values for sodium.

As a result of the spacings of the energy levels, spectra of the outermost electrons of heavy atoms are observed in the visible and ultraviolet regions of the electromagnetic spectrum. Spectra involving the most tightly bound electrons of heavy atoms (e.g. $n = 1$), are observed in the X-ray region of the electromagnetic spectrum.

SUMMARY OF SECTION 3

1 The hydrogen atom was modelled as a particle in a cubic box. In the model, the mass of the particle is taken to be 9.11×10^{-31} kg (approximately the same as that of the electron) and each side of box is taken to be of length 5.00×10^{-10} m (approximately the same as the diameter of the hydrogen atom).

2 The model gave insights into the energy of the electron in the hydrogen atom. First, the energy is *quantized* because the electron is confined. Second, the differences between the electron's lowest few energy levels (roughly a few electronvolts) are determined principally by the size of the atom, by the mass of the electron and by the value of Planck's constant.

3 The differences between the lowest energy levels of electrons in heavy atoms (i.e. atoms that contain more than one electron) are greater than the corresponding differences for the energy levels of atomic hydrogen.

4 Models based on the Schrödinger equation allow electronic energy levels and wavefunctions for atoms to be predicted with high accuracy.

SAQ 3 In this question, you are asked to comment on the formulation and application of the particle-in-a-box model of the hydrogen atom used in this Section.

(a) Why did we formulate the model? Why didn't we use a more complete description of the hydrogen atom?

(b) Wasn't it ridiculous to use a model that is based on the assumption that the hydrogen atom has the shape of a cube?

(c) The prediction of the model for the energy levels of the electron in the hydrogen atom is certainly not in perfect agreement with experiment (Figure 20). Does this disagreement prove that the model is worthless?

SAQ 4 In a nutshell, what were the two key insights about the hydrogen atom that were derived from the particle-in-a-box model?

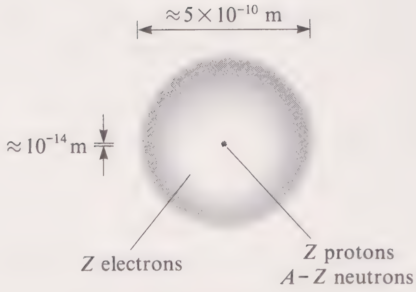


FIGURE 24 An atom contains Z electrons, Z protons and $A - Z$ neutrons, where Z is the atomic number of the nucleus and A is its mass number.

4 THE ATOMIC NUCLEUS

Atomic nuclei were first discussed in Units 11–12, where you saw that the **nucleus** is the tiny, positively charged core of the atom. Apart from the hydrogen nucleus $^1_1$H, which consists of a single proton, nuclei generally consist of both protons and neutrons (Figure 24).

In this Section, several aspects of basic nuclear physics will be discussed. First, the material on nuclei that you met in Units 11–12 will be revised. Second, we shall consider the type of interaction that holds together the protons and neutrons in nuclei. Third, a particle-in-a-box model will be used to try to understand why the energy spacings of the lowest energy levels of nuclei are very much wider than those of the electron in the hydrogen atom. The energy required to break apart atomic nuclei will then be considered, and the Section is concluded by a discussion of nuclear masses in which Einstein's equation $E = mc^2$ is introduced.

4.1 THE CONTENTS OF NUCLEI

The diameter of each nucleus is approximately 10^{-14} m (a hundredth of a millionth of a millionth of a metre), which is very small indeed compared with the typical diameter of an atom, approximately 5×10^{-10} m (Figure 24). The nucleus occupies proportionately about as much space in an atom as a pinhead inside Wembley stadium.

The two types of particle in the nucleus—the proton and the neutron—have approximately the same mass, which is very roughly 1 800 times that of the electron. Hence, most of the mass of each atom is 'concentrated' in its nucleus, which consequently has a much higher density than the atom as a whole. Experiments show that the density of a nucleus is normally about 10^{14} times the density of water. This implies that if a 5 ml teaspoonful of nuclei could be prepared, it would have a mass of about 10^{12} kg—about the mass of Ben Nevis!

In order to specify the numbers of protons and neutrons in a nucleus, two numbers are needed—its mass number and its atomic number (Figure 24). The **mass number** A of a nucleus is defined as the total number of protons and neutrons that it contains: this number gives the number of particles in the nucleus. The **atomic number** Z of the nucleus is defined as the number of protons that it contains. Hence, the number of neutrons in a nucleus is given by the difference $A - Z$ between its mass number and its atomic number (Figure 24).

ISOTOPE

Because the neutron has no electrical charge and the proton has an electrical charge of $+e$ (where e is the magnitude of the charge of the electron), the total charge of the nucleus is $+Ze$. Hence, in an atom, this nuclear charge balances the equal and opposite charge $-Ze$ of the Z electrons that orbit the nucleus (Figure 24).

You may remember from Units 11–12 that atoms whose nuclei contain the same number of protons but different numbers of neutrons have the same chemical properties. This is because these properties depend on the number of electrons in the atom, which in turn depends only on the number of protons in the nucleus.

☐ Can you remember the name given to atoms whose nuclei contain the *same* number of protons but *different* numbers of neutrons?

■ Such atoms are known as **isotopes**; their nuclei have the same atomic number Z, but have different mass numbers.

The contents of the nucleus of an element are usually noted down by writing before its chemical symbol, the mass number of the nucleus (written as a superscript) and its atomic number (written as a subscript). For example, the isotope of silicon with $A = 27$ and $Z = 14$ is written ${}^{27}_{14}\mathrm{Si}$.

ITQ 7 Consider the isotope ${}^{68}_{30}\mathrm{Zn}$ of the element zinc.

(a) How many protons does the nucleus contain?

(b) How many neutrons does it contain?

(c) Given that the electrical charge of the electron is approximately -1.602×10^{-19} C, what is the charge of the nucleus?

(d) How many electrons move around the nucleus of a neutral atom of zinc?

(e) A nucleus X has atomic number 31. Is X an isotope of zinc?

Each element has a characteristic number of unstable isotopes (i.e. isotopes that sooner or later undergo radioactive decay) and a number of completely stable isotopes which never undergo radioactive decay. For example, magnesium has five unstable isotopes—${}^{20}_{12}\mathrm{Mg}$, ${}^{21}_{12}\mathrm{Mg}$, ${}^{23}_{12}\mathrm{Mg}$, ${}^{27}_{12}\mathrm{Mg}$ and ${}^{28}_{12}\mathrm{Mg}$—but only three stable isotopes—${}^{24}_{12}\mathrm{Mg}$, ${}^{25}_{12}\mathrm{Mg}$ and ${}^{26}_{12}\mathrm{Mg}$. Each stable isotope of an element occurs naturally with a characteristic relative abundance on Earth. For example, the three stable isotopes of magnesium ${}^{24}_{12}\mathrm{Mg}$, ${}^{25}_{12}\mathrm{Mg}$ and ${}^{26}_{12}\mathrm{Mg}$ occur naturally on Earth with relative abundances of approximately 78.6%, 10.1% and 11.3%, respectively.

In Figure 25, the number of protons in each of the completely *stable* nuclei that exist in nature is plotted against the number of neutrons that they each contain. Hence, Figure 25 shows all the stable isotopes found in nature.

ITQ 8 How many completely stable isotopes are there of (a) chlorine, which has atomic number 17; (b) technetium, which has atomic number 43?

Notice from Figure 25 that *stable* isotopes whose atomic numbers are less than about 17 contain roughly the *same* number of neutrons as protons. However, stable isotopes with a greater atomic number contain *more* neutrons than protons. The reason for this phenomenon is connected with the electrical charges of the two particles. If neutrons were 'added' to a nucleus, no electrostatic force would act on them because they have no electrical charge. However, if protons were 'added' to a nucleus they would be repelled by the other protons in the nucleus, because the other protons are of course also positively charged (remember, like charges repel). Hence, it is easier to 'add' neutrons to a nucleus than it is to 'add' protons to it. You might think that it should be possible to 'add' as many neutrons as we like to a nucleus, but this turns out not to be possible in practice—if a nucleus contains too many neutrons it is unstable and undergoes radioactive decay.

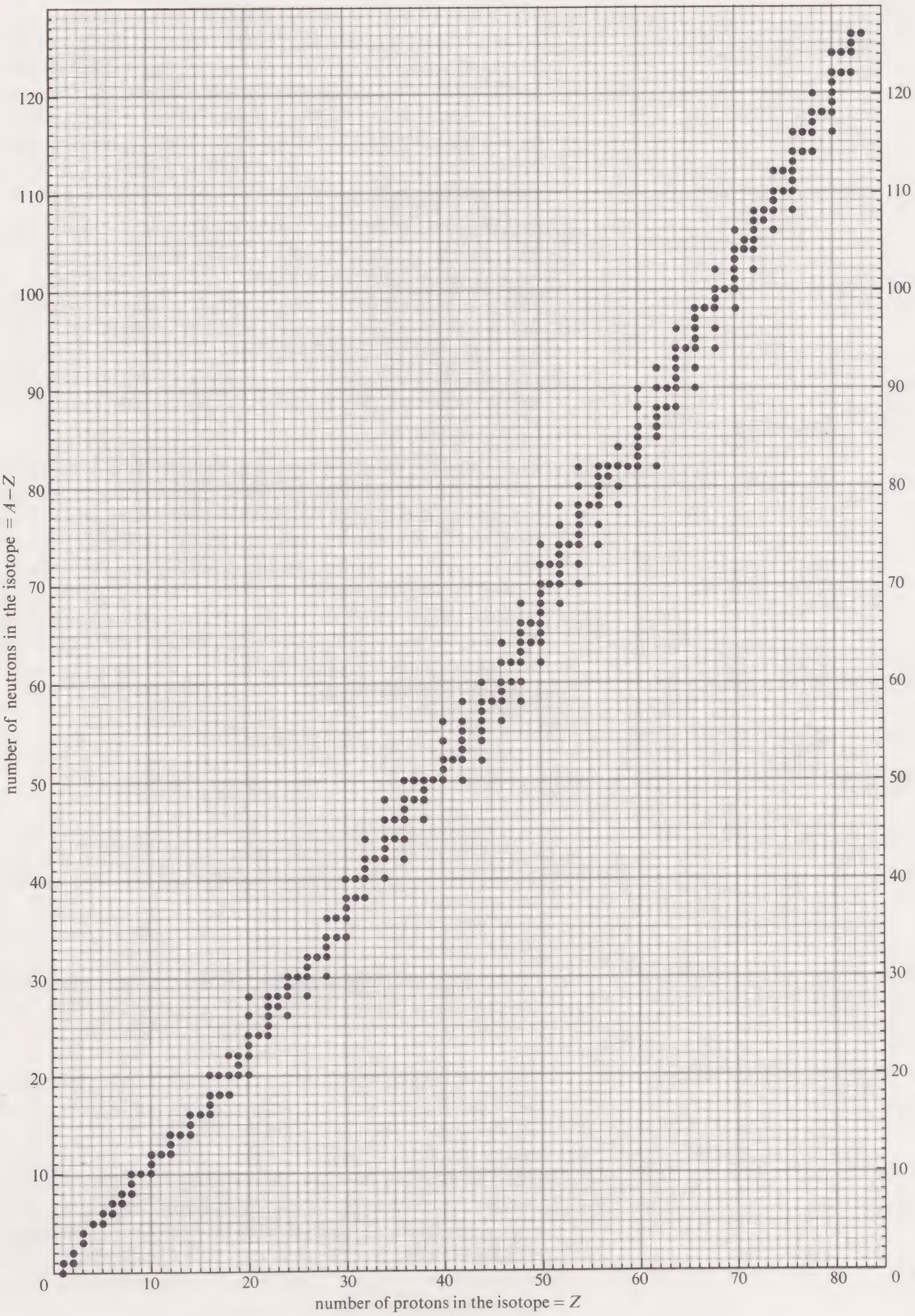


FIGURE 25 The number of protons in *completely stable* nuclei plotted against the number of neutrons that they contain.

4.2 THE STRONG INTERACTION

Why do protons and neutrons bind together to form nuclei? So far in this Course you have met three basic types of interaction that you may think could be responsible for this effect—the gravitational interaction (Unit 3), the electrostatic interaction (Unit 9) and the magnetic interaction (Units 5–6).

STRONG INTERACTION

The gravitational interaction is felt by the protons and neutrons in the nucleus but this interaction is far too weak to bind the particles together. The electrostatic interaction cannot be responsible for the binding: because neutrons have no charge, they are not bound electrostatically to each other or to the protons in the nucleus. Also, because the electrostatic force between particles with charges of the same sign is repulsive, the electrostatic force between the protons pushes them apart! It can also be shown that the magnetic interaction is not responsible for the binding.

The interaction that is responsible for the binding of the protons and neutrons in the nucleus is known as the **strong interaction**. This type of interaction has not been mentioned so far in the Course, mainly because its effects are not directly encountered in everyday life, unlike those of the much more familiar gravitational, electrostatic and magnetic interactions. The strong interaction is actually quite special in the sense that it is felt only by certain particles, for example by the proton and neutron. It is *not* felt by electrons. (We shall discuss the strong interaction in more detail in Unit 32.)

The strong interaction between protons and neutrons has four principal characteristics (Figure 26). First, it is always predominantly attractive, whether it acts between positively charged protons or between uncharged neutrons or between protons and neutrons. This is to be expected, since this interaction binds *both* types of particle to form a nucleus. The second characteristic is that the strength of the interaction is *independent* of the electrical charges and masses of the proton and neutron. This implies that the strength of the strong interaction between two neutrons is exactly the same as that between two protons and that between a neutron and a proton (all at the same separation).

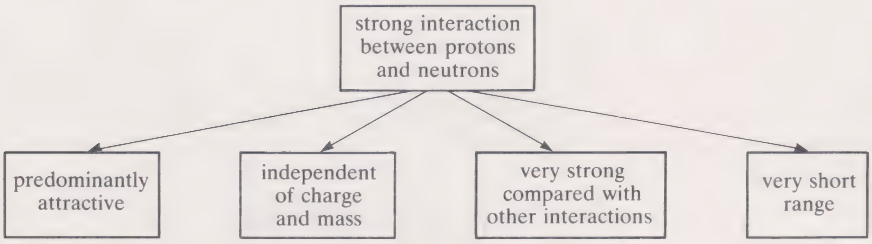

strong interaction between protons and neutrons
predominantly attractive
independent of charge and mass
very strong compared with other interactions
very short range

FIGURE 26 Four characteristics of the strong interaction between protons and neutrons.

The third characteristic of the strong interaction is that its strength is enormous compared with the strengths of electrostatic, magnetic and gravitational interactions. For example, the attractive strong interaction between two protons in a nucleus is very much greater than the repulsive electrostatic interaction between them—this is why they stay bound tightly in the nucleus. Finally, the strong interaction acts only over a short range, approximately 10^{-14} m. Hence, two neutrons separated by 10^{-15} m are attracted to each other by the strong interaction whereas if they were separated by, say, 10^{-12} m the strength of the strong interaction between them would be negligible. This short-range property of the strong interaction helps to explain why nuclei normally have a diameter of approximately 10^{-14} m—within this range, the protons and neutrons are tightly bound to each other by the attractive strong interaction, but outside the range the strength of the interaction is negligible.

There is an important difference between our understanding of the strong interaction and our understanding of the gravitational, magnetic and electrostatic interactions. Whereas the latter three types of interaction are described by comparatively simple laws that have been checked experimentally (e.g. the electrostatic and gravitational interactions are described in terms of Coulomb's law and Newton's gravitational law respectively), no such simple law is known for the strong interaction between protons and neutrons.

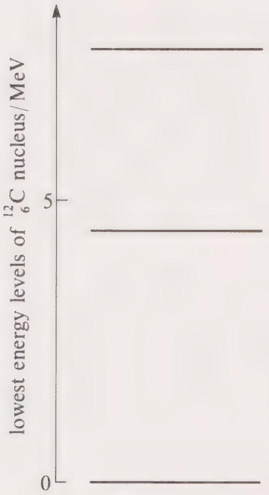


FIGURE 27 The three lowest energy levels of a $^{12}_{6}$C nucleus.

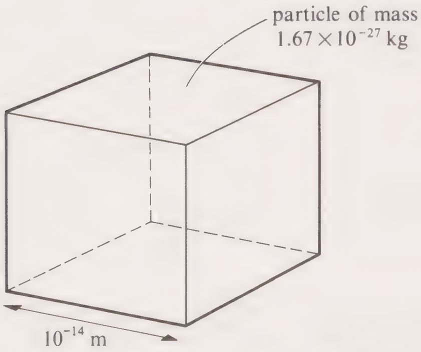


FIGURE 28 A simple model of an atomic nucleus.

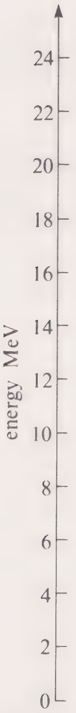


FIGURE 29 See ITQ 11.

4.3 UNDERSTANDING NUCLEAR ENERGY LEVELS (TV programme)

You saw in the Introduction that nuclei have quantized values of energy, known as energy levels (e.g. Figure 27). The spacings of the lowest energy nuclear energy levels are normally approximately a million times those of the lowest few energy levels of the electron in a hydrogen atom, so they are measured in megaelectronvolts (MeV). In this Section, we shall try to understand this by investigating the quantization of nuclear energy.

The investigation will be carried out by modelling a typical nucleus as a cubic box that contains a single particle (Figure 28). Each side of the box will be taken to be 10^{-14} m (a typical nuclear diameter) and the mass of the confined particle will be taken to be 1.67×10^{-27} kg (approximately the mass of the proton and neutron). This model has two notable virtues: it explicitly takes into account

- the size of the nucleus
- the mass of each of the particles in the nucleus.

However, the model also has several shortcomings. It is based on the somewhat unrealistic assumption that nuclei are cubic. Also, it ignores the fact that protons and neutrons have been observed experimentally to have spin. But perhaps the most potentially dangerous assumption of the model is that the details of the strong interaction between a single particle and all the other nuclear constituents are ignored. This assumption is sweeping and apparently extremely unreasonable—we shall not attempt to justify it theoretically, and shall simply go ahead and implement it and then examine the results. If the model is hopelessly unsuccessful, the assumption will obviously have to be modified.

We now want you to use the nuclear particle-in-a-box model. Try ITQs 9–11.

ITQ 9 (a) What is the quantum mechanical equation that gives the possible energy values E of a particle of mass m confined in a cubic box whose sides each have length L? (You will probably need to refer to Section 2.)

$E =$

(b) Use your answer to part (a) to complete the box below.

Protons and neutrons in nuclei have energy levels because they are

------------------------------ .

ITQ 10 (a) Refer to the equation in the answer to part (a) of ITQ 9. Evaluate, in units of joules and to three significant figures, the quantity $h^2/(8mL^2)$ for the nuclear particle-in-a-box model (Figure 28).

(b) Using your answer to part (a) and the definition of the megaelectronvolt MeV (1 MeV $\approx 1.602 \times 10^{-13}$ J), evaluate $h^2/(8mL^2)$ in units of MeV, to two significant figures. Enter your result in the box below.

nuclear particle-in-a-box model

$$E = \quad \text{MeV} \times (n_1{}^2 + n_2{}^2 + n_3{}^2) \qquad (23)$$

ITQ 11 (a) Show on Figure 29 the four lowest nuclear energy levels according to the particle-in-a-box model. (Hint: refer to Figure 15).

(b) Compare the energy spacings of the lowest nuclear energy levels predicted by the particle-in-a-box model (Figure 28) with the experimentally observed nuclear energy levels for carbon shown in Figure 27. How well does the model agree with experiment?

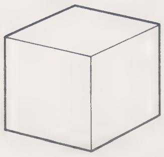
NUCLEAR BINDING ENERGY GRAPH

BINDING ENERGY OF A NUCLEUS

You saw in ITQs 10 and 11 that the nuclear particle-in-a-box model successfully predicts that the lowest nuclear energy levels should be far more widely spaced in energy than those of the electron in a hydrogen atom, as predicted by the atomic particle-in-a-box model. According to this analysis, the spacings of the energy levels of the hydrogen atom and those of the typical nucleus both depend on three quantities—Planck's constant, and the size of the confining volume and the mass of the confined particle in each case. Analyses using more advanced models have confirmed this conclusion.

The two particle-in-a-box models formulated in this Unit—one of the hydrogen atom, the other of a typical nucleus—have been used with considerable success. Although the numerical predictions obtained using the two models have not been very accurate, they have nonetheless allowed qualitative answers to be given to questions Q1–Q4 (about atomic and nuclear energy levels) which were posed in the Introduction. The results we have obtained using the two models are reviewed in Table 2.

TABLE 2 Comparison between the particle-in-a-box models of the hydrogen atom and the typical nucleus

	Hydrogen atom	Typical nucleus
visualization		
model		
energy levels according to the quantum mechanical model	$E = 1.5\,\text{eV} \times (n_1{}^2 + n_2{}^2 + n_3{}^2)$	$E = 2.1\,\text{MeV} \times (n_1{}^2 + n_2{}^2 + n_3{}^2)$
questions answered qualitatively using the model	Q1 Why do atomic electrons have energy levels?	Q3 Why do protons and neutrons in nuclei have energy levels?
	A1 Because the electrons are *confined.*	A3 Because the protons and neutrons are *confined.*
	Q2 Why are the differences between the lowest energy levels of the electron in a hydrogen atom a few electronvolts?	Q4 Why are the differences between the lowest energy levels of protons and neutrons in nuclei approximately a million times those of the electron in a hydrogen atom?
	A2 The separations are determined to be a few electronvolts mainly by the size of the atom, by the mass of the electron, and by the value of Planck's constant.	A4 The differences in the spacing arise principally from the differences between (i) the sizes of the atom and the nucleus and (ii) the masses of the confined particles involved.

4.4 NUCLEAR BINDING ENERGIES

It is found experimentally that the minimum energy required to break up a nucleus into free protons and neutrons is normally in the range 1–2 000 MeV. Such experiments determine the **binding energy of a nucleus**, which is actually the minimum energy required to unbind the nucleus. When it is said that the 'binding energy of an oxygen nucleus $^{16}_{8}O$ is 127.6 MeV', it is meant that 127.6 MeV of energy must be transferred to an $^{16}_{8}O$ nucleus in order to break it up into eight free protons and eight free neutrons with zero kinetic energy (Figure 30).

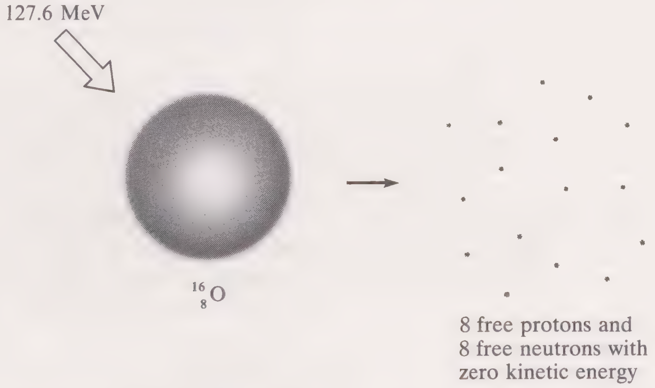


FIGURE 30 At least 127.6 MeV of energy must be transferred to a $^{16}_{8}O$ nucleus in order to unbind completely its constituent protons and neutrons.

If the mass number *A* of a nucleus and its total binding energy are known, it is easy to calculate the average amount of energy required to unbind a *single* proton or neutron from the nucleus:

$$\begin{pmatrix}\text{average binding energy}\\ \text{of a nucleus per}\\ \text{constituent proton}\\ \text{and neutron}\end{pmatrix} = \frac{\begin{pmatrix}\text{minimum energy required to unbind}\\ \text{completely all the protons}\\ \text{and neutrons in the nucleus}\end{pmatrix}}{\text{mass number } A \text{ of the nucleus}} \qquad (24)$$

For example, consider again the oxygen $^{16}_{8}O$ nucleus, which has mass number $A = 16$ and so contains 16 particles (8 protons and 8 neutrons). Because the binding energy of an $^{16}_{8}O$ nucleus is 127.6 MeV,

$$\begin{pmatrix}\text{average binding energy of an}\\ ^{16}_{8}\text{O nucleus per constituent}\\ \text{proton and neutron}\end{pmatrix} = \frac{127.6\,\text{MeV}}{16} \approx 7.98\,\text{MeV}$$

In Figure 31, the **nuclear binding energy graph**, the average binding energy of a nucleus per constituent proton and neutron is plotted against mass number for many nuclei. You should check that the point for $^{16}_{8}O$ (calculated above) has been plotted correctly on this graph.

There are three features that you should note about Figure 31.

- The graph has a maximum around the points that correspond to $^{56}_{26}Fe$ and $^{62}_{28}Ni$, whose average binding energy per constituent proton and neutron is 8.79 MeV. The constituents of these nuclei are more tightly bound than those of any other nucleus.
- The *average* energy required to unbind a proton or neutron from any of the nuclei with mass number *A* greater than 10 is approximately 8 MeV.
- The amounts of energy required to unbind a proton or neutron from certain nuclei are markedly higher than those immediately closest to them in mass number—in other words, there are certain nuclei that are exceptionally strongly bound for their mass numbers. Three such nuclei are $^{16}_{8}O$, $^{12}_{6}C$, and $^{4}_{2}He$ (an α-particle).

SPECIAL THEORY OF RELATIVITY

EINSTEIN'S EQUATION

REST MASS

4.5 NUCLEAR MASSES AND EINSTEIN'S EQUATION

In 1905, Einstein formulated the **special theory of relativity**, a revolutionary theory of space and time. The most important result of the theory for our purposes is that *an amount of energy (of any type) has an equivalent mass.* More specifically, Einstein predicted that the mass m that is equivalent to energy E is given by

$$m = E/c^2 \tag{25}$$

where c is the speed of light in a vacuum, approximately $3 \times 10^8\,\mathrm{m\,s^{-1}}$. Equation 25 is better known in the form

$$\boxed{E = mc^2 \qquad (26)}$$

which is generally known as **Einstein's equation.** The statement 'an amount of energy has an associated mass' implies that if the energy of a particle changes its mass must also change. This is correct, but it appears to lead to a problem—how can the mass of a particle be defined uniquely? The way out of this is simply to define the **rest mass** of any entity, which is the mass of the entity as measured by an observer *relative to whom the entity is at rest*. The masses of the electron, proton and neutron quoted on the back of the physics and general science Units are actually the *rest* masses of the particles. (The rest mass of the photon turns out to be zero.)

Why did hundreds of years elapse between the invention of the concept of energy and Einstein's discovery of the connection between energy and mass? Well, consider the mass that is equivalent to a joule of energy, roughly the energy required to lift an apple by 1 m on Earth: this amount of energy is, within a few orders of magnitude, typical of the energies normally encountered in everyday life. Equation 25 says that the mass that is equivalent to a joule of energy is given by

$$\begin{aligned}\text{mass equivalent to 1 J of energy} &= \frac{1\,\mathrm{J}}{(3 \times 10^8\,\mathrm{m\,s^{-1}})^2}\\ &\approx 10^{-17}\,\mathrm{kg}\end{aligned}$$

i.e. 0.000 000 000 000 000 01 kg, much too small to be measured using conventional weighing instruments! In view of this, it is hardly surprising that the connection between energy and mass was not discovered in everyday experiments—it took the genius of Einstein for the connection to be *predicted*.

For more than twenty years after Einstein proposed his equation, it could not be checked, mainly because the energies involved in chemical reactions are associated with masses that are much too small to be observed. However, in the 1930s, when it became possible to study nuclear reactions, the validity of Einstein's equation was clearly demonstrated. In order to understand why the consequences of the equation are comparatively easy to observe in the case of nuclei, think again about the example illustrated in Figure 30, which shows an oxygen nucleus $^{16}_{8}\mathrm{O}$ that is broken up into its free constituents when 127.6 MeV of energy have been transferred to it. Remember that 127.6 MeV is the *minimum* energy required to unbind the nucleus, so after this energy has been transferred the unbound constituents have zero kinetic energy.

The situation shown in Figure 30 can be expressed by writing down a kind of energy equation

$$^{16}_{8}\mathrm{O}\ \text{nucleus} + 127.6\,\mathrm{MeV} \longrightarrow \begin{array}{l}\text{8 free protons and 8 free neutrons}\\ \text{with zero kinetic energy}\end{array}$$

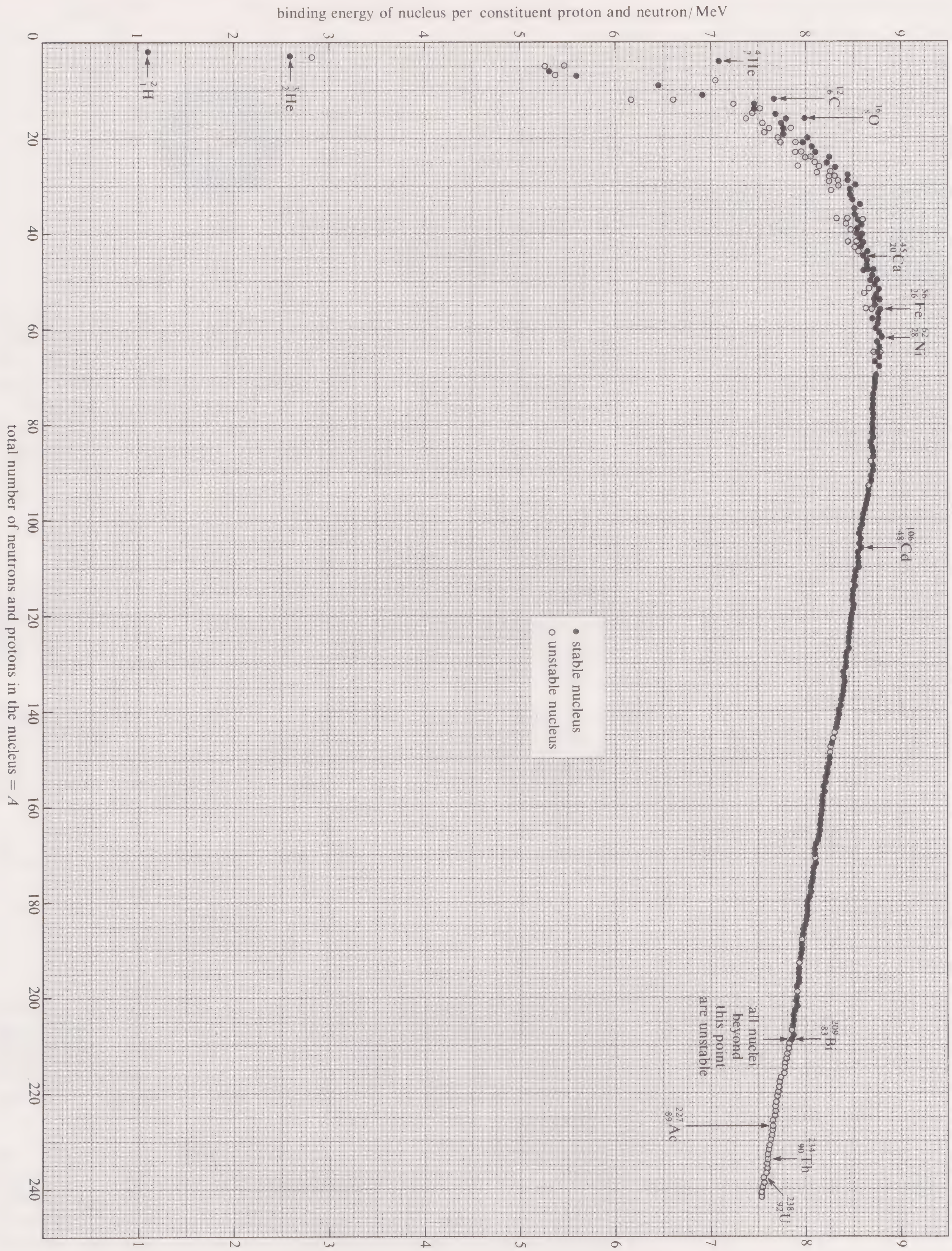


FIGURE 31 The nuclear binding energy graph.

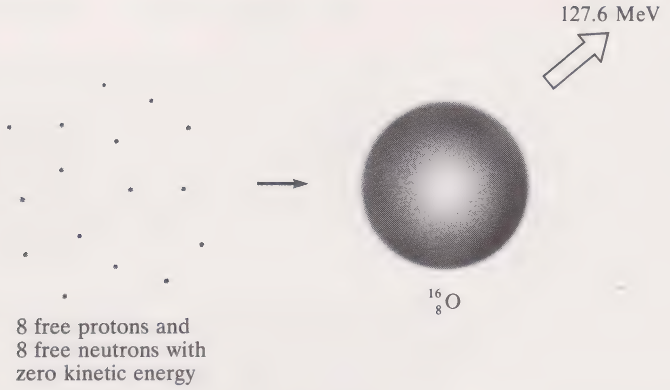


FIGURE 32 When 8 free protons and 8 free neutrons with zero kinetic energy coalesce to form a $^{16}_{8}O$ nucleus, 127.6 MeV of energy are released (cf. Figure 30).

This 'equation' also implies that if the reverse process were to occur (Figure 32), i.e. if the 8 free protons and 8 free neutrons with zero kinetic energy were to coalesce to form the $^{16}_{8}O$ nucleus, then 127.6 MeV of energy would be released (electromagnetic radiation would be emitted with this energy). In order to see this, it is best to write the 'equation' the other way round:

$$\begin{array}{l}\text{8 free protons and 8 free neutrons}\\ \text{with zero kinetic energy}\end{array} \longrightarrow {}^{16}_{8}\text{O nucleus} + 127.6\,\text{MeV}$$

This implies that

$$\begin{array}{l}\text{total rest mass of 8 free protons}\\ \text{and 8 free neutrons}\end{array} = \begin{array}{l}\text{rest mass of}\\ {}^{16}_{8}\text{O nucleus}\end{array} + \begin{array}{l}\text{mass equivalent to}\\ 127.6\,\text{MeV}\end{array}$$

which in turn implies that *the sum of the rest masses of the free protons and neutrons is greater than the rest mass of the nucleus that is formed from them.* How much less than the sum of the rest masses of the free protons and neutrons is the rest mass of the nucleus? Let us calculate the difference using Einstein's equation.

We know the energy released in units of MeV so, in order to calculate the energy's equivalent mass in SI units of kilograms we must first convert the energy into SI units of joules. Because 1 MeV $\approx 1.602 \times 10^{-13}$ J,

$$\begin{aligned}127.6\,\text{MeV} &= 127.6\,\text{MeV} \times (1.602 \times 10^{-13}\,\text{J MeV}^{-1})\\ &= 2.044 \times 10^{-11}\,\text{J}\end{aligned}$$

Hence, using Equation 25 and the value of the speed of light in a vacuum ($c \approx 2.998 \times 10^{8}\,\text{m s}^{-1}$),

$$\begin{aligned}\text{mass equivalent to 127.6 MeV} &= \frac{2.044 \times 10^{-11}\,\text{J}}{(2.998 \times 10^{8}\,\text{m s}^{-1})^2}\\ &= 2.27 \times 10^{-28}\,\text{kg}\end{aligned}$$

Hence, the mass that is equivalent to the energy released is tiny—the sum of the rest masses of the free protons and neutrons is only very slightly greater than the rest mass of the nucleus that is formed from them. But the crucial point is that this mass difference is a significant and *measurable* fraction (approximately 0.8%) of the total rest mass of the free constituents (approximately 2.68×10^{-26} kg).

Experiments have confirmed this prediction of Einstein's equation and, moreover, the principle used in this discussion of the $^{16}_{8}O$ nucleus applies equally well to all other nuclei that contain more than one constituent: *the rest mass of a nucleus that contains more than one constituent is* less *than the sum of the rest masses of its free constituents, because when the constituents coalesced to form the nucleus, energy was released.* This applies not only to nuclei—it applies to *any* group of particles that are bound together. For example, a hydrogen chloride molecule HCl, which consists of a hydrogen atom H bound to a chlorine atom Cl, has a rest mass that is *less* than the sum of the rest masses of a free hydrogen atom and a free chlorine atom. The mass difference is, however, only approximately three hundredths of a millionth of a per cent of the rest mass of the molecule—far too small to be measured using conventional weighing apparatus.

This illustrates the general point that the difference between the rest mass of a bound system and the sum of the rest masses of its free constituents is always proportionately much larger for nuclei than it is for molecules (and atoms). This is why Einstein's equation is much more important in nuclear physics than it is in chemistry.

SUMMARY OF SECTION 4

1 Each atom has a nucleus—a positively charged 'core' that has a 'diameter' of approximately 10^{-14} m. Apart from the hydrogen nucleus ^1_1H, which is a proton, nuclei in general consist of combinations of protons and neutrons.

2 Each nucleus is characterized by its atomic number Z (which specifies the number of protons it contains) and its mass number A (which specifies the total number of its constituent protons and neutrons). Different isotopes of an element are characterized by different mass numbers but the same atomic number.

3 Each type of nucleus has only a certain number of unstable isotopes (which sooner or later undergo radioactive decay) and a certain number of completely stable isotopes (which never undergo radioactive decay). There are a finite number of completely stable nuclei (Figure 25).

4 Protons and neutrons in nuclei are bound together by the strong interaction, whose four principal characteristics are given in Figure 26.

5 A particle-in-a-box model can be used to understand why nuclei have energy levels and why the energy spacings of the lowest of these levels are approximately a million times the energy spacings of the lowest energy levels of the electron in a hydrogen atom.

6 The binding energy graph (Figure 31) shows that the average binding energy of a nucleus per constituent proton and neutron is approximately 8 MeV for most nuclei.

7 The special theory of relativity tells us that an amount of energy has an equivalent mass. The mass m that is equivalent to energy E is given by Einstein's equation $E = mc^2$, where c is the speed of light in a vacuum. The equation leads to an understanding of why the rest mass of each nucleus that contains more than one constituent particle is *less* than the sum of the rest masses of its free constituents.

SAQ 5 Which two of the following statements about the interactions between particles in a nucleus are correct?

(a) Protons and neutrons in the nucleus are not subject to a gravitational interaction.

(b) The particles in a nucleus are bound together by the strong interaction.

(c) The magnitude of the strong interaction between two neutrons separated by 10^{-16} m is less than the magnitude of the strong interaction between two protons at the same separation.

(d) The magnitude of the strong interaction between two neutrons that are separated by 1 cm is negligible.

SAQ 6 (a) The principal interactions that are felt by protons in a nucleus are strong and electrostatic. Are these interactions respectively attractive or repulsive?

(b) Figure 25 indicates that after a certain atomic number ($Z = 83$) no nuclei are completely stable. Use your answer to part (a) to suggest why. (Hint: think about the interactions of the constituent protons).

RADIOACTIVE DECAY

α-DECAY

β^--DECAY

SAQ 7 The minimum energy required to break apart the stable beryllium nucleus $^{10}_{4}Be$ into free protons and neutrons is approximately 64.98 MeV.

(a) What is the average binding energy of the $^{10}_{4}Be$ nucleus per constituent proton and neutron? Mark the value on the nuclear binding energy graph (Figure 31.)

(b) Are the constituents of a $^{10}_{4}Be$ nucleus more or less tightly bound than those of a $^{12}_{6}C$ nucleus?

SAQ 8 Refer to Figure 31.

(a) What is the *minimum* energy required to unbind the constituents of a $^{4}_{2}He$ nucleus (i.e. an α-particle)?

(b) How much energy is released when an α-particle is produced from two free protons and two free neutrons?

(c) What is the difference between the rest mass of an α-particle and the sum of the rest masses of two free protons and two free neutrons?

5 RADIOACTIVE DECAYS OF NUCLEI

You saw in the previous Section that only certain nuclei are completely stable (Figure 25), and that there are many unstable nuclei, which sooner or later undergo **radioactive decay**. This Section concerns types of radioactive decay that can occur, and we shall also discuss briefly why the products of radioactive decay can be dangerous to life. (We shall not be discussing the half-lives of the radioactive nuclei, as that subject was considered in detail earlier in the Course, notably in Units 11–12 and 28–29.)

You have met the phenomenon of radioactivity several times in this Course. We mentioned it in Units 7–8 and 27, where you saw that the energy released in radioactive decays is mainly responsible for the very high temperatures of the Earth's interior and, ultimately, for the energy that drives the massive lithospheric plates in the plate tectonic process. You also saw in Units 11–12 how the ages of objects such as biblical documents can be estimated by measuring the amount of $^{14}_{6}C$ that they contain—this is known as carbon dating. Finally, you saw in Units 28–29 how the ages of certain minerals can be determined from a knowledge of the decay rates of their radioactive constituents.

5.1 α-DECAY

When a nucleus undergoes **α-decay**, a helium nucleus $^{4}_{2}He$ (α-particle) is ejected and another nucleus is formed. The mass number of the other nucleus is four less and its atomic number is two less than the corresponding values of the original nucleus. An example of α-decay is provided by the decay of a stationary nucleus of $^{238}_{92}U$ (Figure 33):

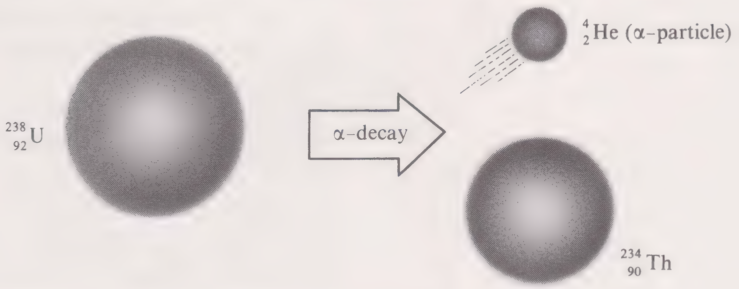


FIGURE 33 Schematic visualization of the α-decay of a $^{238}_{92}U$ nucleus.

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He \text{ (α-particle)} \qquad (27)$$

It is found experimentally that the total kinetic energy of the products is approximately 4.3 MeV, nearly all of which is carried by the α-particle—in other words, the energy released in the reaction is approximately 4.3 MeV. (We say that energy is released in a reaction when the total kinetic energy of the particles before the reaction is less than the kinetic energy afterwards.) The rest mass of the uranium nucleus must therefore be greater than the sum of the rest masses of the thorium and helium nuclei. Using the law of conservation of energy, and assuming that the uranium nucleus decays from rest,

$$\begin{array}{l}\text{energy equivalent}\\ \text{to rest mass of}\\ {}^{238}_{92}\text{U}\end{array} = \begin{array}{l}\text{energy equivalent}\\ \text{to rest mass of}\\ {}^{234}_{90}\text{Th}\end{array} + \begin{array}{l}\text{energy equivalent}\\ \text{to rest mass of}\\ {}^{4}_{2}\text{He}\end{array} + \begin{array}{l}\text{total kinetic}\\ \text{energy of}\\ {}^{234}_{90}\text{Th and } {}^{4}_{2}\text{He}\end{array} \tag{28}$$

☐ By how much does the rest mass of the uranium nucleus exceed the sum of the rest masses of the thorium and helium nuclei?

■ Because 4.3 MeV of energy are released in the reaction, Equation 28 tells us that the energy equivalent to the rest mass of the uranium nucleus is greater than the sum of the energies due to the rest masses of the thorium nucleus and the helium nucleus by 4.3 MeV. It follows that the rest mass of the uranium nucleus is greater than the sum of the rest masses of the thorium and helium nuclei by the mass that is equivalent to 4.3 MeV:

$$\begin{aligned}\text{mass equivalent to 4.3 MeV} &= 4.3\,\text{MeV}/c^2\\ &= 4.3\,\text{MeV} \times \frac{1.6 \times 10^{-13}\,\text{J}\,\text{MeV}^{-1}}{(3.0 \times 10^{8}\,\text{m}\,\text{s}^{-1})^2}\\ &\approx 7.6 \times 10^{-30}\,\text{kg}\end{aligned}$$

As you can see from Figure 34, the constituents of the thorium nucleus are more tightly bound than those of the original uranium nucleus. This always happens in α-decay—the heavy nucleus formed is always more tightly bound than was the original nucleus.

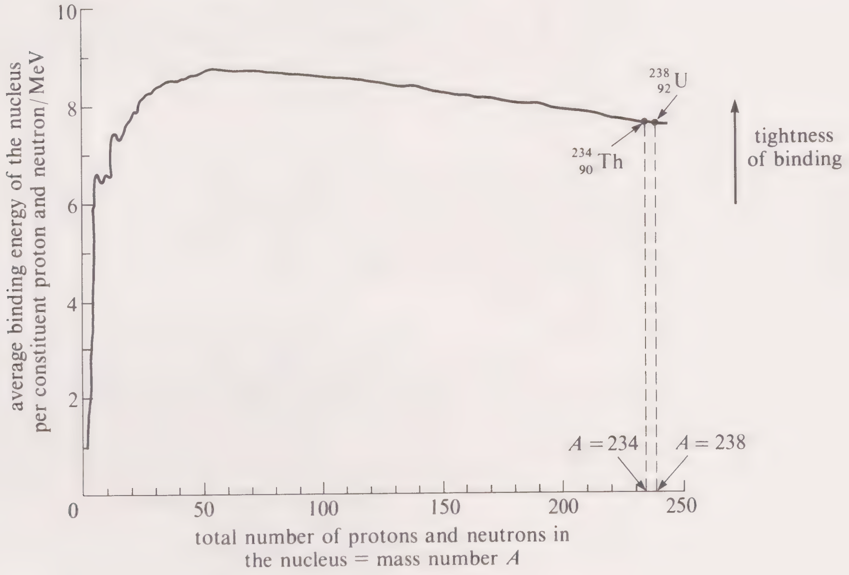


FIGURE 34 Sketch of the nuclear binding energy graph (Figure 31), with the values for the ${}^{238}_{92}\text{U}$ and ${}^{234}_{90}\text{Th}$ marked on.

5.2 β^--DECAY

You saw in Units 11–12 that in **β^--decay** a neutron in an unstable nucleus transforms into a proton with the emission of an electron. Thus, the nucleus formed after the decay contains one more proton than the original nucleus, but both nuclei contain the same total number of particles. Hence, the nucleus formed in β^--decay has an atomic number that is greater by one than that of the original nucleus, and both nuclei have the same mass number. An example of a nucleus that undergoes β^--decay is ${}^{15}_{6}\text{C}$:

$${}^{15}_{6}\text{C} \longrightarrow {}^{15}_{7}\text{N} + {}^{0}_{-1}\text{e}^- \tag{29}$$

β^+-DECAY

γ-DECAY

EXCITED NUCLEUS

Now it is time to admit that the account that we gave of β^--decay in Units 11–12 was incomplete. In this type of decay, a particle known as an electron antineutrino $\bar{\nu}_e$ is also emitted. This particle, which will be discussed further in Unit 32, has no electrical charge, zero atomic number and zero mass number.

Equation 29 should therefore be rewritten as

$$^{15}_{6}\mathrm{C} \longrightarrow {}^{15}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e}^- + \bar{\nu}_e \qquad (30)$$

You should now cross out Equation 29 to remind yourself that it is not correct! The process that occurs in a $^{15}_{6}\mathrm{C}$ nucleus that undergoes β^--decay is the transformation of a neutron n into a proton p, an electron and an electron antineutrino $\bar{\nu}_e$. This is the basic process of β^--decay:

$$\boxed{\text{basic } \beta^-\text{-decay process} \quad \mathrm{n} \longrightarrow \mathrm{p} + \underbrace{\mathrm{e}^- + \bar{\nu}_e}_{\text{ejected}} \qquad (31)}$$

The β^--decay of $^{15}_{6}\mathrm{C}$ (Equation 30) is illustrated schematically in Figure 35.

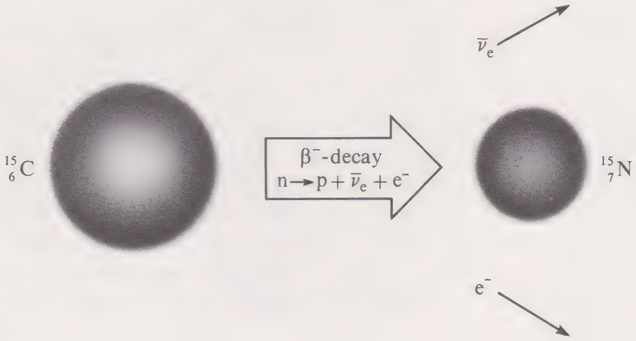


FIGURE 35 Schematic visualization of the β^--decay of a $^{15}_{6}\mathrm{C}$ nucleus.

It is found experimentally that in this particular decay, the total kinetic energy of the ejected electron and the electron antineutrino is of the order of a megaelectronvolt, 1 MeV. This is typical of the energy released in β^--decay.

☐ Given that energy is released in β^--decay, is the rest mass of the original nucleus equal to the sum of the rest masses of the nucleus and other particles produced in the decay?

■ No. The difference between the rest mass of the original nucleus and the sum of the rest masses of the nucleus and the other particles produced is equal to the mass that is equivalent to the energy released. (The reasoning that leads to this conclusion is identical with the reasoning used following Equation 28, in the analogous case of α-decay.)

5.3 β^+-DECAY

In **β^+-decay**, a proton in the original nucleus transforms into a neutron and two particles that are ejected, a positron e^+ and an electron neutrino ν_e:

$$\boxed{\text{basic } \beta^+\text{-decay process} \quad \mathrm{p} \longrightarrow \mathrm{n} + \underbrace{\mathrm{e}^+ + \nu_e}_{\text{ejected}} \qquad (32)}$$

You have met neither the e^+ nor the ν_e particle so far in the Course but you will become more familiar with them in Unit 32. For the moment, all you need to know about them is that both have zero mass number, and that the positron has the same positive charge as that of the proton whereas the electron neutrino has no charge. These properties imply that the nucleus formed after β^+-decay has the same mass number as the original nucleus, whereas the atomic number of the nucleus formed is one less than that of the original nucleus.

An example of a nucleus that undergoes β^+-decay is $^{11}_{6}\text{C}$: after the decay, the stable boron nucleus $^{11}_{5}\text{B}$ is formed (Figure 36):

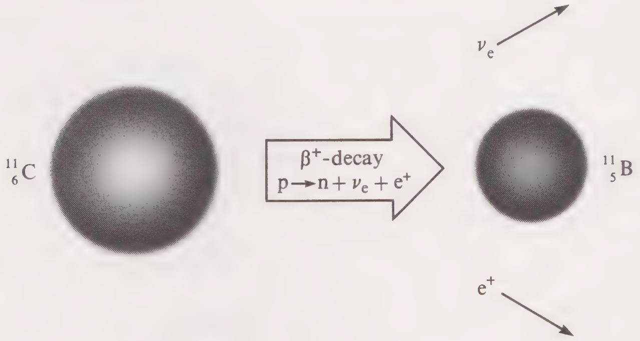


FIGURE 36 Schematic visualization of the β^+-decay of a $^{11}_{6}\text{C}$ nucleus.

$$^{11}_{6}\text{C} \longrightarrow {}^{11}_{5}\text{B} + {}^{0}_{1}\text{e}^{+} + \nu_e \qquad (33)$$

The energy released in this β^+-decay process (and in most others) is of the order of a megaelectronvolt.

5.4 γ-DECAY

The process of **γ-decay** is the simplest radioactive decay process to understand, because it is exactly analogous to the emission of a photon of electromagnetic radiation after an electron in an atom in an excited state makes a transition to a lower energy level.

The process of γ-decay usually occurs after an **excited nucleus**—a nucleus with an energy greater than its lowest possible energy— is produced in an α- or β-decay. When a proton or neutron in an excited nucleus makes a transition to a lower energy level, a photon of electromagnetic radiation is emitted with an energy equal to the difference in energy of the two levels involved.

☐ Bearing in mind what you know about the differences between the lowest energy levels of nuclei, what do you expect to be a reasonable estimate of the energy of a photon emitted from an excited nucleus?

■ Roughly of the order of *millions* of electronvolts, MeV. The energy of the photon is equal to the difference between the two energy levels involved. As you saw in Section 4, this difference is roughly of the order of megaelectronvolts.

A photon with an energy of the order of megaelectronvolts is called a γ-ray—that, of course, is why the process of γ-ray emission is called γ-decay. It would be reasonable to object that the process is not really a decay but a *transition*, so it would perhaps be better to call the process a γ-transition. However, γ-decay is the term that is used conventionally to describe the process, so we shall continue to use it here.

NUCLEAR DECAY CHAIN

NUCLEAR DECAY CHANNEL

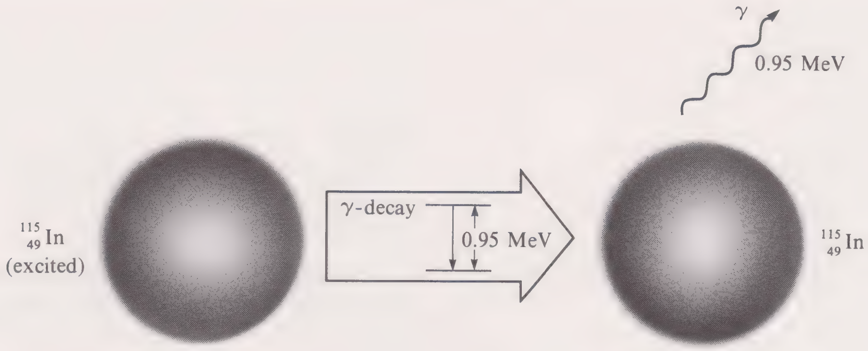


FIGURE 37 Schematic visualization of the γ-decay of an excited $^{115}_{49}$In nucleus.

An example of γ-decay is illustrated in Figure 37. In this case, a γ-ray photon is emitted with an energy of 0.95 MeV when an excited indium nucleus $^{115}_{49}$In makes a transition to an energy level that is lower by 0.95 MeV:

$$\underset{\text{excited nucleus}}{^{115}_{49}\text{In}} \longrightarrow {^{115}_{49}\text{In}} + \underset{\text{ejected photon}}{\gamma} \tag{34}$$

☐ Is the rest mass of the excited $^{115}_{49}$In nucleus, greater than, less than or equal to the rest mass of the $^{115}_{49}$In nucleus left after the γ-decay?

■ Because total energy is conserved, the energy that is equivalent to the rest mass of the excited nucleus is *greater* than the energy equivalent to the rest mass of the nucleus left after the γ-decay:

$$\begin{array}{l}\text{energy equivalent}\\ \text{to the rest mass}\\ \text{of excited } {^{115}_{49}\text{In}}\end{array} = \begin{array}{l}\text{energy equivalent}\\ \text{to the rest mass of}\\ {^{115}_{49}\text{In}} \text{ left after } \gamma\text{-decay}\end{array} + \begin{array}{l}0.95\,\text{MeV energy of}\\ \text{the photon}\end{array}$$

Hence, the difference between the rest mass of the excited $^{115}_{49}$In nucleus and the rest mass of the $^{115}_{49}$In nucleus is the mass that is equivalent to the 0.95 MeV energy (approximately 1.7×10^{-30} kg).

We ought to point out that α-, β- and γ-decays are not the only kinds of radioactivity—there are other ways in which nuclei can spontaneously decay. For example, in 1984 two physicists at Oxford University discovered a new (and very rare) type in which the radium nucleus $^{223}_{88}$Ra decays into a lead nucleus $^{209}_{82}$Pb with the ejection of a $^{14}_{6}$C nucleus! Also, as you will see in Section 6, there is a process called spontaneous nuclear fission, which should also be classified as a type of radioactive decay.

5.5 DECAY CHAINS

Most unstable nuclei do *not* decay in one step: the majority of them decay via several steps before a completely stable one is formed. For example, consider the decay $^{238}_{92}\text{U} \rightarrow {^{206}_{82}\text{Pb}}$, which you met in connection with geological dating in Units 28–29. After the decay, the mass number of the original nucleus has decreased by 32 and its atomic number has decreased by 10. Clearly, the uranium nucleus has not undergone just one α-, β^-- or β^+-decay; the nucleus has undergone several decays in what is usually called a **nuclear decay chain**. You can trace for yourself the 'links' in the chain using Figure 38.

The Figure shows that the $^{238}_{92}$U nucleus first undergoes α-decay to form the nucleus $^{234}_{90}$Th, which then undergoes β^--decay to form a $^{234}_{91}$Pa nucleus. And so the decays go on until, finally, the completely stable lead nucleus $^{206}_{82}$Pb is formed. One interesting point about the decay chain shown in Figure 38 is that when the nucleus $^{214}_{83}$Bi is formed, it can undergo *either* α-decay to form $^{210}_{81}$Tl *or* β^--decay to form $^{214}_{84}$Po—the $^{214}_{83}$Bi nucleus is said to have two **nuclear decay channels**.

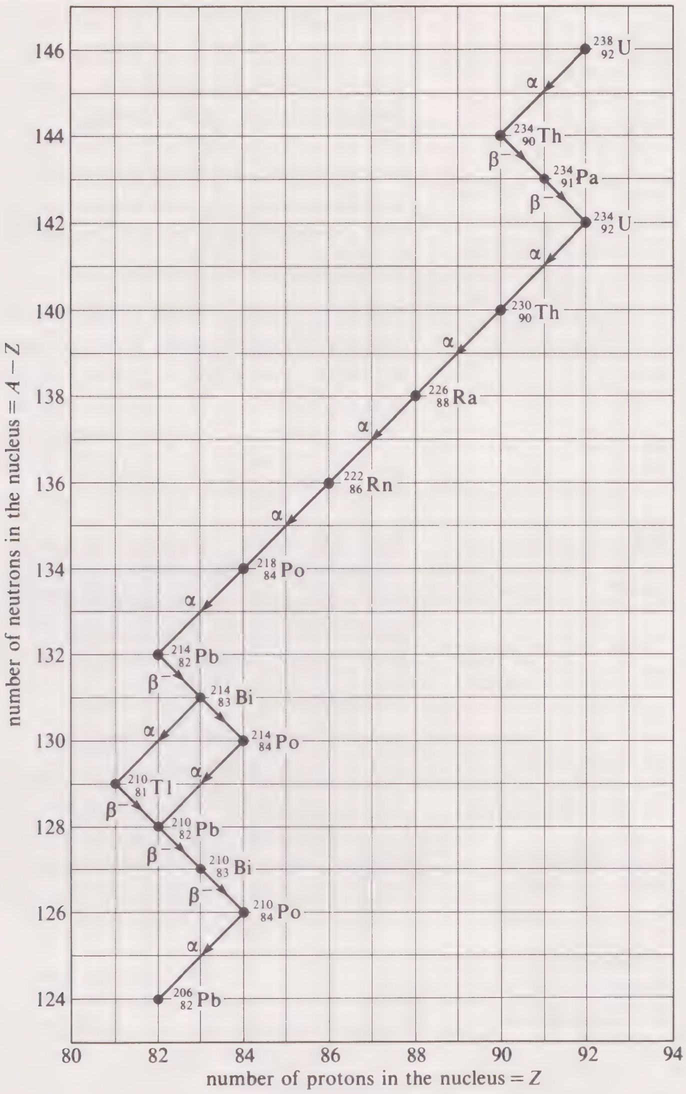


FIGURE 38 The decay chain $^{238}_{92}U \longrightarrow ^{206}_{82}Pb$.

5.6 HAZARDS OF RADIOACTIVITY

Most people are aware that the products of radioactive processes can be hazardous. The after-effects of the dropping of the nuclear bombs on Japan in 1945 and of the more recent tragedy at Chernobyl (Figure 39) chillingly bear witness to the possible dangers. But *why* are the products of radioactive processes potentially hazardous to living organisms? And to what extent are these products dangerous in everyday life? We shall discuss these questions briefly in this Section.

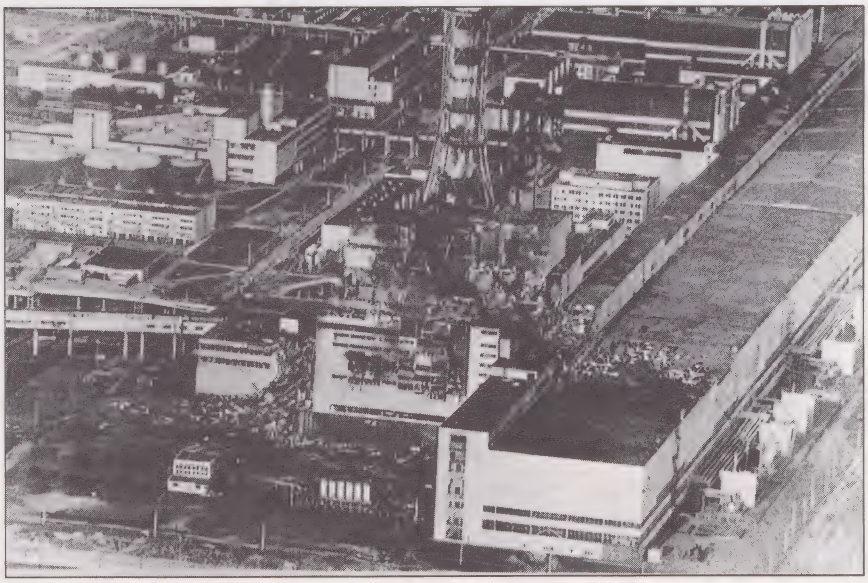


FIGURE 39 On 26 April 1986, at 01.23 a.m., one of the plants at the Chernobyl nuclear power station exploded when the nuclear reactions in the plant went out of control. At the subsequent enquiry, it was found that the accident was due to the failure of the engineers at the plant to observe standard safety procedures.

IONIZING RADIATION

Radioactive processes are potentially dangerous because their products—such as α-, β- and γ-particles—normally have sufficient kinetic energy to ionize matter (in particular, living tissue) with harmful effects. These ionizing products—and all other particles that can ionize matter—are generally known as **ionizing radiation**, and each type of this radiation has a characteristic depth of penetration into tissue (Table 3). Also, each type of ionizing radiation has a characteristic ionization density for living tissue, that is, each type causes a characteristic number of ionizations per unit length of its track (Figure 40).

TABLE 3 Ranges of some types of ionizing radiation

Type of ionizing radiation	Range in tissue
α-particles	10^{-5} to 10^{-4} m
β-particles	10^{-3} to 10^{-2} m
γ- and X-rays	through body

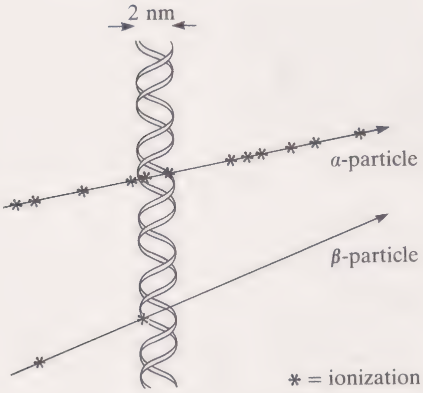


FIGURE 40 An α-particle with an energy of a few megaelectronvolts is much more likely than a β-particle to damage both strands of a DNA molecule. In each single ionization shown here, approximately 33 eV of energy are transferred. (Note that we have not shown the track of a γ-particle on this Figure as no ionizations would occur on the scale of the Figure: γ-radiation is even more sparsely ionizing than β-particles.)

The interactions of the ionizing radiation with water are very important for human beings because about 80% of the body is water. The ions produced in these interactions can subsequently damage molecules of DNA, the genetic material which is contained inside the nuclei of the body's cells. If a DNA molecule is broken it may rejoin, but the repair may be incomplete or incorrect. There may then be a change in the structure of the DNA molecule and therefore a change in the genetic message.

Sometimes, the death of a cell occurs as a result of radiation-induced damage to a DNA molecule in the cell's nucleus. However, surviving cells that still contain damaged DNA may become transformed. If damage occurs at gene locations involved in cell regulation, then cancer can result. In a gamete, the breaking of the DNA may lead to inherited disorders in the organism's descendants.

Breaks in the DNA strands can result in chromosome structural aberrations that may activate cancer-causing genes (oncogenes). These aberrations may be observed during mitosis soon after the initial radiation damage has occurred, when they may appear, for example, as misjoined breaks or fragments (Figure 41). Table 4 summarizes the observable biological effects of ionizing radiation and the times at which they occur after the radiation damage originally took place.

The amount of ionizing radiation required to kill an organism varies considerably from one organism to another. Most mammals, including *Homo sapiens*, will die within a month or two of receiving a dose of radiation in which the energy absorbed by the tissue is roughly $10\,\mathrm{J\,kg^{-1}}$. Most insects can survive doses of up to several hundred joules per kilogram. For comparison, in the biology experiment at Summer School, the irradiated wheat seedlings were exposed to a radiation dose of about $5\,000\,\mathrm{J\,kg^{-1}}$!

It is worth noting that the effects of a dose of ionizing radiation on an organism depend on the time over which the dose is delivered. For example, for someone who receives a dose of $5\,\mathrm{J\,kg^{-1}}$ in a few minutes, there is a chance of roughly 50% of death within two months from damage to the

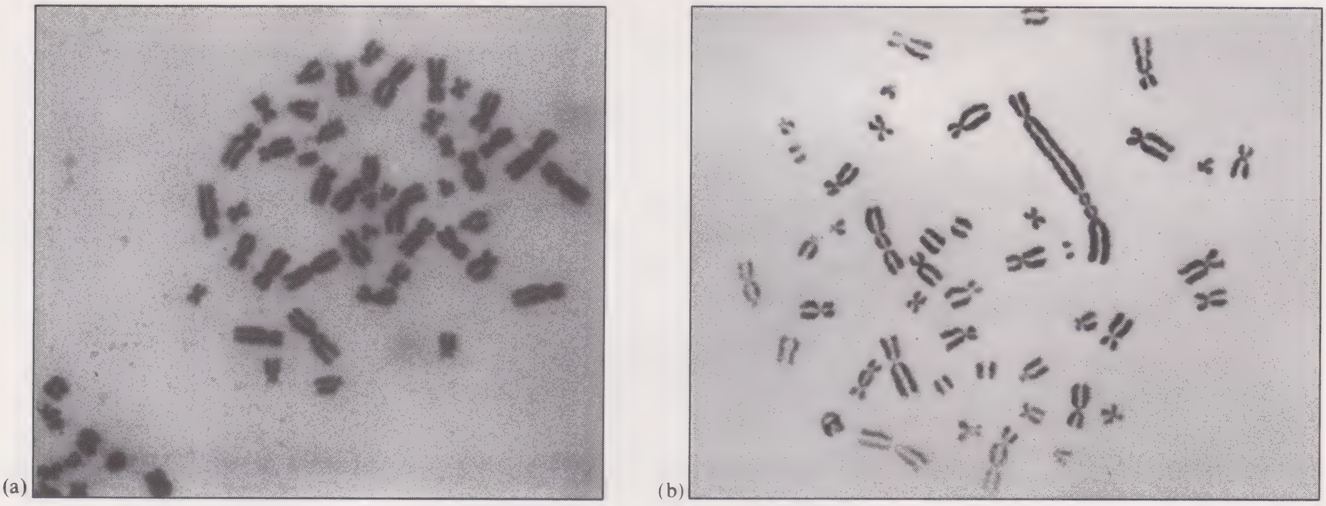


FIGURE 41 The biological effects of radiation. Both pictures show chromosome preparations from white blood cells of a human male. (a) Chromosomes from unirradiated blood, in which the chromatids of each pair are attached to each other only at the centromere. (b) In chromosomes from irradiated blood, abnormal structures can be seen. These result from the repair of damaged molecules being incomplete or incorrect. (Photos courtesy of (a) Berkeley Nuclear Laboratories and (b) National Radiological Protection Board.)

TABLE 4 Some ionizing radiation effects in mammals and the timescale over which the effects occur

Biological level or event	Important effects of ionizing radiation	Timescale
ionization	production of radicals (i.e. reactive molecules) and excited molecules	10^{-14} to 10^{-3} seconds
molecular	damage to macromolecules (e.g. enzymes, RNA and DNA); interference with metabolic pathways	seconds to hours
subcellular	damage to cell membranes, nucleus, chromosomes, mitochondria etc.	seconds to days
cellular	inhibition of cell division; cell transformation; cell death	minutes to years
tissue and organ	disruption of central nervous system; cell death in the gastro-intestinal tract and in bone marrow; cancer induction	minutes to decades
animal	death or life-shortening	minutes to decades
animal populations	genetic changes due to gene and chromosome mutations in individuals	months to generations

bone marrow. However, if the same dose is delivered over several years, the chance of death is roughly 10%, and in this case death would result from long-term effects, such as cancer.

You may well be wondering what dose of ionizing radiation you receive from day to day. The *average* annual dose for people living in the United Kingdom is approximately $2.2 \times 10^{-3}\,\mathrm{J\,kg^{-1}}$, and this dose is the result of contributions from several sources, as you can see in Figure 42. This Figure shows that by far the greatest contribution to the total average dose comes from natural background sources (which correspond to the shaded part of the Figure). These sources include cosmic rays from outer space (Unit 32), and the radioactive processes that occur within the Earth and in surface rocks and soil. In addition, we are all exposed to ionizing radiation from natural and artificial radioactive nuclei that enter our bodies through the food we eat and the air we breathe. Some of these nuclei are potentially more harmful than others if they are concentrated or retained in tissues that are particularly sensitive to the effects of radiation, for example the bone marrow and the lungs. In the UK, probably the most important single cause of lung cancer is (apart from smoking) radon gas, which is produced from the decay of uranium in rocks and soil.

It is worth stressing that Figure 42 applies to the *average* annual dose for people living in the UK. For someone who works with artificial sources of ionizing radiation, the ratio of artificial to natural background radiation is likely to be substantially higher than it is in the average case. For example, the average occupational dose received by workers in nuclear power stations in the UK in recent years has been about 65% of the natural background dose.

It is asserted in some quarters that the artificial background is dangerously high for many people and that the risks associated with unnatural ionizing radiation sources are unacceptable. On the other hand, it can be argued that the risks are not significantly greater than those associated with other aspects of modern life, for example, with road travel. The risks associated with ionizing radiation are extremely difficult to quantify, and they are regularly reviewed by bodies such as the International Commission on Radiological Protection (ICRP).

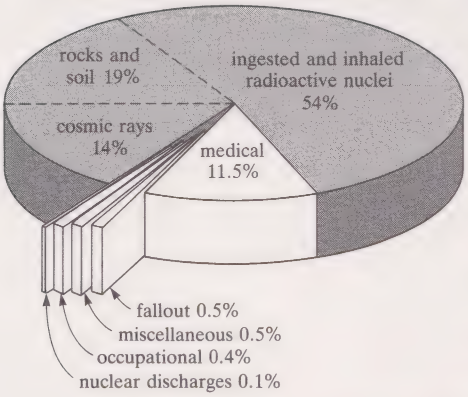


FIGURE 42 Approximate contributions to the dose of ionizing radiation received *on average* by people living in the UK. (Source: National Radiological Protection Board, 1984).

NUCLEAR FISSION

SPONTANEOUS NUCLEAR FISSION

SUMMARY OF SECTION 5

1 In the process of α-decay, a nucleus decays to form a lighter nucleus with the emission of an α-particle ($^4_2$He nucleus).

2 In the process of β^--decay, a neutron in the original nucleus transforms into a proton, emitting an electron and an electron antineutrino.

3 In the process of β^+-decay, a proton in the original nucleus transforms into a neutron, emitting a positron and an electron neutrino.

4 In the process of γ-decay, a proton or neutron in an excited nucleus makes a transition to a lower energy level, and a γ-ray photon of electromagnetic radiation is emitted.

5 Most unstable nuclei do not decay in one step to form stable nuclei. For example, the unstable nucleus $^{238}_{92}$U decays via several intermediate unstable nuclei before the completely stable nucleus $^{206}_{82}$Pb is formed (Figure 38).

6 Ionizing radiation can be hazardous to living organisms. For the majority of people living in the UK, the largest component of their total ionizing radiation dose is from natural background sources. However, for some groups of the population, the ratio of artificial to natural background ionizing radiation is substantially higher than average.

SAQ 9 A stationary nucleus of polonium $^{212}_{84}$Po undergoes α-decay and one of the decay products is a nucleus of lead (Pb).

(a) Write down the equation for this decay.

(b) Find an expression for the total kinetic energy of the lead nucleus and the α-particle in terms of the rest masses of the nuclei involved in the reaction and the speed of light in a vacuum.

SAQ 10 The isotope of calcium that contains 17 neutrons decays to form a potassium nucleus with mass number 37 and atomic number 19. Of what type of radioactive decay process is this an example? Write down the equation that describes the decay.

SAQ 11 The process of radioactive γ-decay is analogous to the process in which visible light is emitted from certain atoms. In what way(s) are the processes similar, and in what way(s) are they different?

SAQ 12 Which one of the following statements is correct?

(a) The term ‘ionizing radiation’ is synonymous with electromagnetic radiation.

(b) When ionizing radiation damages a DNA molecule in the nucleus of a cell, the damage is always permanent.

(c) For the majority of people living in the UK, their average annual dose of ionizing radiation is mainly due to natural background sources.

(d) The only known effect of ionizing radiation on human beings is the induction of cancer.

6 NUCLEAR FISSION AND FUSION

In this Section, we shall consider nuclear fission and fusion, and we shall briefly discuss nuclear power.

6.1 NUCLEAR FISSION

In the previous Section, you saw that when a nucleus undergoes α-decay, an α-particle is ejected, and a new and more tightly bound nucleus is formed. Energy is released because the rest mass of the original nucleus is greater than the sum of the rest masses of the nuclei formed. There is a similar type of process, known as **nuclear fission**, in which a heavy nucleus decays into two comparatively light nuclei of roughly equal rest mass, with the release of energy.

An example of this process is provided by a possible decay channel of the nucleus of fermium $^{256}_{100}\mathrm{Fm}$, in which it undergoes **spontaneous fission** into a xenon nucleus $^{140}_{54}\mathrm{Xe}$, a palladium nucleus $^{112}_{46}\mathrm{Pd}$, and four neutrons:

$$^{256}_{100}\mathrm{Fm} \longrightarrow {}^{140}_{54}\mathrm{Xe} + {}^{112}_{46}\mathrm{Pd} + 4{}^{1}_{0}\mathrm{n} \qquad (35)$$

(Remember that the neutron has mass number $A = 1$ and atomic number $Z = 0$, so it is denoted by $^{1}_{0}\mathrm{n}$.) The nuclear binding energy graph (Figure 43) shows that the constituents of the xenon nucleus and those in the palladium nucleus that are formed after the fission process are each more tightly bound than the constituents of the original nucleus.

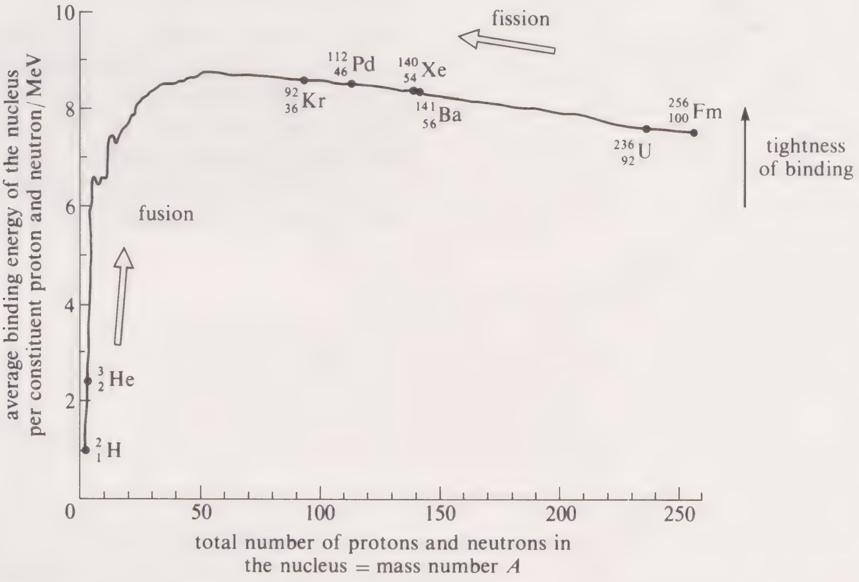


FIGURE 43 The points that correspond to the nuclei that take part in the fission and fusion reactions discussed in Section 6 shown on a sketch of the nuclear binding energy graph (Figure 31).

The rest mass of the fermium nucleus is greater than the sum of the rest masses of the products by approximately 2.67×10^{-28} kg, which corresponds to an energy of approximately 150 MeV. Hence, the total kinetic energy of the products is approximately 150 MeV:

$$^{256}_{100}\mathrm{Fm} \longrightarrow \underbrace{{}^{140}_{54}\mathrm{Xe} + {}^{112}_{46}\mathrm{Pd} + 4{}^{1}_{0}\mathrm{n}}_{\text{total kinetic energy of approximately 150 MeV}} \qquad (36)$$

We say that the *nuclear energy* released in the reaction is approximately 150 MeV. An important point to bear in mind about this process is that the only nuclei that frequently undergo this type of decay are the heaviest, artificially produced nuclei such as $^{256}_{100}\mathrm{Fm}$, which we have just discussed. The vast majority of nuclei much more frequently undergo the other types of spontaneous radioactive decay (α, β and γ) that we have described in Section 5.

INDUCED NUCLEAR FISSION

NUCLEAR CHAIN REACTION

NUCLEAR FUSION

A more common type of fission occurs when energy is transferred to certain heavy nuclei by bombarding them with particles, such as protons, neutrons and α-particles. An example is the fission of the uranium nucleus ${}^{235}_{92}\mathrm{U}$, which hardly ever undergoes spontaneous fission but can be *induced* to undergo fission by bombarding it with neutrons*. If a ${}^{235}_{92}\mathrm{U}$ nucleus captures one of these neutrons, it can form an *unstable* nucleus of uranium.

☐ What are the mass number and the atomic number of the unstable nucleus formed when the uranium nucleus ${}^{235}_{92}\mathrm{U}$ captures a neutron?

■ The mass number of the original nucleus increases by one, so, for the nucleus formed, $A = 236$; the atomic number of the nucleus formed is the same as the original one, so $Z = 92$.

The unstable uranium nucleus ${}^{236}_{92}\mathrm{U}$ can decay in many different ways or, in other words, this nucleus has many decay channels. One channel through which it decays frequently is the one in which a nucleus of barium ${}^{141}_{56}\mathrm{Ba}$ is formed together with a nucleus of krypton ${}^{92}_{36}\mathrm{Kr}$ and three neutrons.

$$\underset{\text{incident neutron}}{{}^{1}_{0}\mathrm{n}} + {}^{235}_{92}\mathrm{U} \longrightarrow \underset{\text{unstable uranium nucleus}}{{}^{236}_{92}\mathrm{U}} \longrightarrow \underbrace{{}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}}_{\text{total kinetic energy of approximately 200 MeV}} \tag{37}$$

The nuclear energy released in this **induced nuclear fission** reaction is approximately 200 MeV, which is enormous compared with the energy of the products of a typical chemical reaction. For example, the energy of the products in a single nuclear fission reaction is roughly a million times the chemical energy released in the combustion of a molecule of the octane used in the engine of a motor car.

An important feature of the fission reaction described by Equation 37 is that the neutrons that are released can be used again to bombard other ${}^{235}_{92}\mathrm{U}$ nuclei, which can beget yet more neutrons. As you can see from Figure 44, as more nuclei of uranium undergo fission in the **chain reaction**, so more and more energy is released in the form of kinetic energy of the products. It is in this way that the vast amounts of energy are released in the explosion of nuclear bombs.

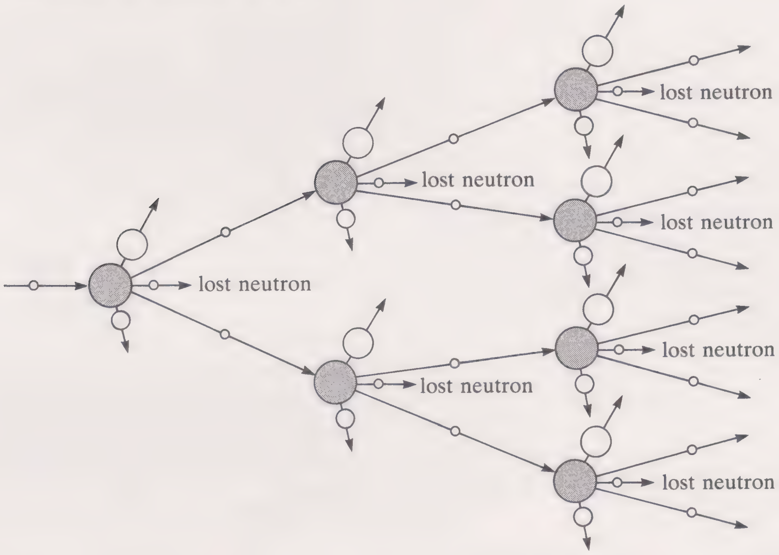


FIGURE 44 Schematic visualization of the induced fission of a ${}^{235}_{92}\mathrm{U}$ nucleus. If two of the released neutrons individually collide with ${}^{235}_{92}\mathrm{U}$ nuclei and the two ${}^{236}_{92}\mathrm{U}$ nuclei formed subsequently undergo fission, more energy will be released and so the process can proceed as a self-perpetuating, uncontrolled chain reaction. This is the type of reaction that underlies the working of a nuclear bomb.

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* It turns out that the induced fission process is most likely to occur if the ${}^{235}_{92}\mathrm{U}$ nucleus is bombarded by neutrons that have a very small amount of kinetic energy (roughly 0.03 eV).

6.2 NUCLEAR FUSION

There is another type of nuclear reaction that is in a sense the opposite process to nuclear fission. In this other type of reaction, **nuclear fusion**, two light nuclei fuse to form a comparatively heavy nucleus, and energy is released.

As an example of nuclear fusion, consider the reaction in which two hydrogen nuclei $^{2}_{1}\mathrm{H}$ fuse to form a nucleus of helium $^{3}_{2}\mathrm{He}$, with the emission of a neutron. In this reaction, the difference between the total rest mass of the two original hydrogen nuclei and the total rest mass of the helium nucleus and neutron produced, is such that the nuclear energy released is approximately 3.3 MeV:

$$^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \longrightarrow \underbrace{^{3}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}}_{\substack{\text{total kinetic}\\ \text{energy approx.}\\ 3.3\,\text{MeV}}} \qquad (38)$$

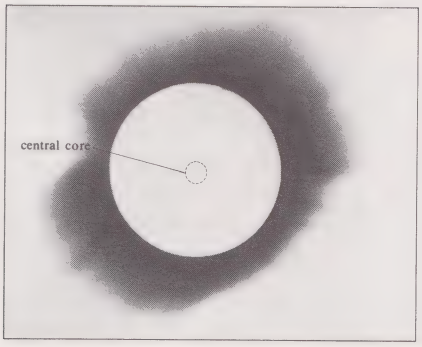


FIGURE 45 Nuclear fusion reactions take place in the Sun's central core. The energy released in these reactions fuels the planet Earth.

Nuclear fusion is an extremely important process. It is responsible for most of the energy we use on Earth, because a high proportion of this energy is derived from nuclear fusion reactions that take place in the Sun's central core (Figure 45). Just think, all the energy you will use in your life is derived from nuclear reactions that took place 93 million miles away!

Nuclear fusion is the process that underlies the operation of hydrogen bombs, the first of which was exploded in 1952. However, it has proved more difficult to use the process for peaceful means, as you will see in Section 6.3.

6.3 NUCLEAR POWER

> The energy produced by the breaking down of the atom is a very poor kind of thing. Anyone who expects a source of power from the transformation of these atoms is talking moonshine.
>
> Ernest Rutherford, addressing the British Association at Leicester on 11 September 1933.

Within ten years of Rutherford's speaking those immortal words, the world's first nuclear reactor was built (Figure 46), and by 1956 energy generated in nuclear fission reactions was being supplied to the UK's national grid. Currently, about 20% of the electrical energy produced in the United Kingdom is generated by nuclear reactors (Figure 47). Each of these reactors uses nuclear fission reactions to generate energy, and all of them use uranium as their fuel.

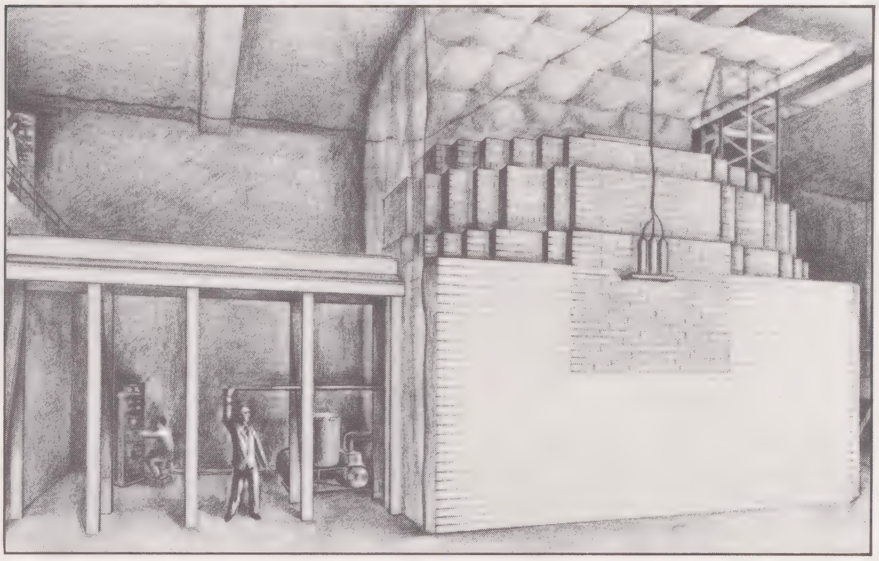


FIGURE 46 A sketch of the world's first nuclear reactor (because of wartime secrecy, photography was not encouraged). A self-sustaining nuclear chain reaction was first achieved on 2 December 1942 in an improvized laboratory in the squash court under the west stands of the University of Chicago's Stagg Field stadium. The leader of the team who built the reactor was the Italian physicist Enrico Fermi.

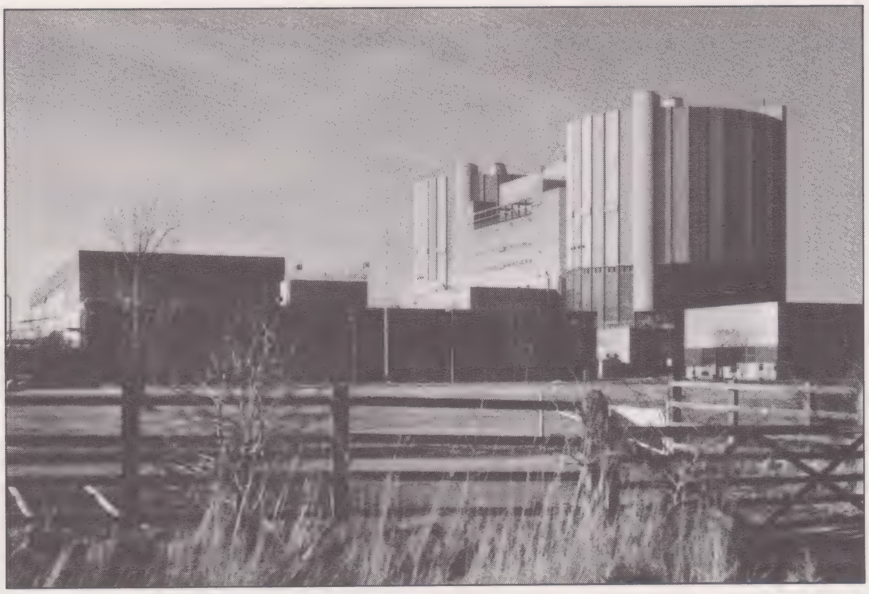
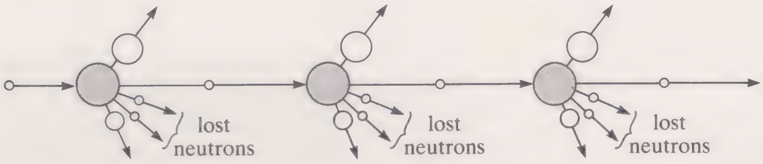


FIGURE 47 The nuclear power station at Oldbury-on-Severn in Avon. (Photo courtesy of United Kingdom Atomic Energy Authority.)

Only 0.7% of naturally occurring uranium is the ^{235}U isotope, and the remaining 99.3% is ^{238}U. This is unfortunate from the nuclear engineer's point of view because the ^{238}U isotope almost never undergoes fission—only the comparatively rare ^{235}U is useful as nuclear fuel. For this reason, reactor fuels are normally enriched artificially so that they contain a few per cent of the ^{235}U isotope.

You have already seen how enormous amounts of energy can be generated in *uncontrolled* nuclear chain reactions—this is the principle behind the nuclear bomb. In a nuclear *reactor*, the fission reaction is carefully controlled by ensuring that only *one* of the neutrons resulting from each fission event collides with a uranium nucleus (Figure 48). The kinetic energy of the products of the reactions is converted into heat, which is in turn converted into electrical energy, which is supplied to the national grid.

FIGURE 48 A *controlled* self-sustaining nuclear chain reaction (cf. Figure 44). This is the type of reaction that underlies the working of nuclear reactors.



The generation of electrical energy from nuclear fission reactions is of course a sensitive issue. Environmental pressure groups argue, for example, that the chances of a reactor going out of control are unacceptably high and that there are serious risks involved with the disposal of the radioactive waste products of nuclear reactions. On the other hand, proponents of nuclear power believe that the risks are acceptably low, and they point to what they believe is the low cost of nuclear energy (compared with, for example, the cost of obtaining energy from fossil fuels). These are bound to remain controversial questions for many years to come.

It is hoped that it will be possible in the future to generate electrical power using energy released in nuclear fusion reactions. Several international teams of researchers are currently investigating the feasibility of generating power from this type of nuclear reaction. This work is being done notably in the United States, the Soviet Union, Japan and Europe.

Three reactions that appear to be particularly promising as a potential source of nuclear energy are

$$^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} \qquad (38)^*$$

$$^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \longrightarrow {}^{3}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \qquad (39)$$

$$^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \longrightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} \qquad (40)$$

One advantage of using fusion reactions such as these to generate energy is that only Reaction 39 produces a radioactive product, ${}^{3}_{1}H$, and that can be used up via Reaction 40. The only other source of radioactivity comes from the interactions of energetic neutrons with surrounding material. Also, the ${}^{2}_{1}H$ fuel for these reactions is available in almost unlimited quantities from the water in our lakes and oceans, and it is very cheap to extract.

These are potentially enormous advantages over the methods of generating power from nuclear fission reactions. However, there are many difficulties associated with the generation of energy from nuclear fusion. First and foremost, it is not easy to force two nuclei to fuse.

☐ Why is it difficult to make two nuclei undergo fusion?

■ Because nuclei are always positively charged, they repel each other electrostatically. In order for the nuclei to fuse, they must be propelled towards each other with sufficient energy to overcome the repulsion.

In order to initiate fusion reactions, the ingredients of the reaction must be heated to a temperature in the region of 100–200 million kelvin—considerably hotter than the central core of the Sun! So, to get the fusion reaction going, a considerable amount of energy must be transferred to the reactants. They must then be kept together at high temperature for long enough to ensure a worthwhile return of energy on the initial investment. Whereas in stars, fusing matter is held together by gravitational fields, scientists on Earth have to try to achieve this effect using magnetic fields.

It is hoped that the intensive research work being done on nuclear fusion will shortly bear fruit—optimists believe that nuclear fusion reactors may be in operation early in the 21st century. If this goal is reached, it would certainly be an enormous achievement—and one that would be of huge benefit to the human race.

SUMMARY OF SECTION 6

1 In the process of nuclear fission, the atomic nucleus that splits almost always has a large mass number, and usually splits into two smaller nuclei and a number of particles, such as neutrons.

2 In the process of nuclear fusion, two light nuclei combine to form one or more other nuclei, and possibly other particles.

3 Nuclear power stations convert energy released in nuclear fission reactions into electrical energy. It is not yet possible to harness energy released in nuclear fusion reactions, but much research is being done to investigate the feasibility of generating power from such reactions.

SAQ 13 (a) Explain briefly the difference between spontaneous nuclear fission and induced nuclear fission.

(b) A uranium nucleus ${}^{236}_{92}U$ spontaneously undergoes fission to form a nucleus of iron (Fe) whose *atomic* number is 26 and a nucleus of dysprosium (Dy) whose *mass* number is 180; there are no other products of the decay. Find the equation that describes this reaction.

SAQ 14 Two helium nuclei ${}^{3}_{2}He$ fuse to form *two* hydrogen nuclei ${}^{1}_{1}H$ and a nucleus of helium ${}^{4}_{2}He$, with the release of energy.

(a) Write down the equation that describes this reaction.

(b) Is the total rest mass of the two ${}^{3}_{2}He$ nuclei equal to the sum of the rest masses of the nuclei formed in the reaction? Explain your answer briefly.

7 WHAT NEXT?

In Units 30 and 31, we have looked at the implications of two revolutionary theories of modern physics—quantum mechanics and relativity.

We have of course concentrated on quantum mechanics. You saw in Unit 30 that this theory concerns phenomena that, in some cases, transcend common sense. Before the theory was formulated, who would have thought that people are diffracted as they walk through doorways? And who would have believed that if you know for certain where something is, you can't possibly know both its speed and the direction in which it is moving? In Unit 31, you have seen that this theory allows fascinating insights into atomic and nuclear physics—in particular, the theory enables an understanding not only of why atoms and nuclei have energy levels, but also of why the differences between hydrogen's energy levels are so much smaller than the differences between the lowest energy levels of nuclei.

Einstein's special theory of relativity was needed to understand why comparatively huge amounts of energy are released in nuclear reactions, such as fission and fusion. What began in 1905 as the remarkably audacious theoretical idea that an amount of energy has an equivalent mass, is now used routinely in, for example, the design of nuclear power stations. Physicists also use the idea as an essential tool in one of the most exciting branches of modern science, particle physics. It is to this subject that we now turn, in the final Unit of the Course.

OBJECTIVES FOR UNIT 31

After you have worked through this Unit, you should be able to:

1 Explain the meaning of, and use correctly, all the terms flagged in the text.

2 Apply the formula $E_n = n^2h^2/(8ml^2)$ for the energy levels of a particle confined in one dimension between parallel plates, and explain why such a particle has energy levels. (*ITQs 1 and 2*)

3 Apply the formula $E = h^2/(8mL^2)\,(n_1{}^2 + n_2{}^2 + n_3{}^2)$ for the energy levels of a particle confined in a cubic box. (*ITQs 3, 5, 6 and 9–11; SAQs 1 and 2*)

4 Formulate and apply particle-in-a-box models of (i) a hydrogen atom and (ii) a typical nucleus, and recognize the limitations of the models. (*ITQs 4–6 and 9–11; SAQs 3 and 4*)

5 Recall that the lowest energy levels of the electron in a hydrogen atom are separated in energy by a few electronvolts, whereas the lowest energy levels of a typical nucleus are separated by millions of electronvolts. (*ITQs 5, 6 and 11; SAQs 3 and 4*)

6 Sketch and interpret the graph of the number of protons in completely stable nuclei plotted against the number of neutrons that they contain. (*ITQ 8; SAQ 6*)

7 Sketch and interpret the nuclear binding energy graph. (*SAQs 7 and 8*)

8 Recall the principal characteristics of the strong interaction between protons and neutrons. (*SAQs 5 and 6*)

9 Interpret and apply (in simple situations) Einstein's equation, $E = mc^2$. (*SAQs 8, 9 and 14*)

10 Recall the basic α, β^-, β^+ and γ radioactive decay processes, and explain why energy is released in each type of process. (*SAQs 9–11*)

11 Summarize, in simple terms, the hazards of ionizing radiation, and recall that for people living in the United Kingdom, the largest component of the annual average dose of ionizing radiation is due to natural background sources. (*SAQ 12*)

12 Interpret equations that describe nuclear fusion and fission, and explain why energy is released in these processes. (*SAQs 13 and 14*)

ITQ ANSWERS AND COMMENTS

ITQ 1 For each possible wavefunction of the confined particle, a whole number of half-wavelengths always 'fit' between the plates (the half-wavelengths are said to be 'quantized'). Because the particle's wavelength and the magnitude of its momentum are related, it follows that the magnitude of its momentum is quantized, and this in turn implies that its energy is quantized.

ITQ 2 (a) Using $m = 1.25 \times 10^{-27}\,\text{kg}$ and $l = 6.63 \times 10^{-5}\,\text{m}$ in conjunction with Equation 11, $E_n = n^2h^2/(8ml^2)$:

$$E_n = \frac{n^2 \times (6.63 \times 10^{-34}\,\text{J s})^2}{8 \times (1.25 \times 10^{-27}\,\text{kg}) \times (6.63 \times 10^{-5}\,\text{m})^2}$$

$$= (10^{-32}\,\text{J}) \times n^2$$

Note how the units work out:

$$(\text{J s})^2/(\text{kg m}^2) = \text{J}^2\,\text{s}^2\,\text{kg}^{-1}\,\text{m}^{-2},$$

and because

$$1\,\text{J} = 1\,\text{kg}\,\text{m}^2\,\text{s}^{-2}$$

(Unit 9), this expression may be expanded to give

$$\text{kg}^2\,\text{m}^4\,\text{s}^{-4}\,\text{s}^2\,\text{kg}^{-1}\,\text{m}^{-2} = \text{kg}\,\text{m}^2\,\text{s}^{-2} = \text{J}.$$

(b) You should use the answer to part (a) to show the $n = 1$ energy level at $E_1 = 10^{-32}\,\text{J}$ (i.e. $10^{-32}\,\text{J} \times 1^2$) and the $n = 3$ energy level at $E_3 = 9 \times 10^{-32}\,\text{J}$ (i.e. $10^{-32}\,\text{J} \times 3^2$).

(c) The energy of the photon emitted in the transition from $n = 3$ to $n = 1$ is $8 \times 10^{-32}\,\text{J}$.

The energy $\Delta E_{3\to1}$ of the photon emitted in the transition is equal to the energy difference between the $n = 3$ energy level and the $n = 1$ energy level. Using the answer to part (a),

$$\begin{aligned}\Delta E_{3\to1} &= E_3 - E_1\\ &= [10^{-32}\,\text{J} \times 3^2] - [10^{-32}\,\text{J} \times 1^2]\\ &= 8 \times 10^{-32}\,\text{J}\end{aligned}$$

(d) The frequency of the emitted radiation is $1.2 \times 10^2\,\text{Hz}$ and its wavelength is $2.5 \times 10^6\,\text{m}$.

Using the standard equation for the energy E of a photon, $E = hf$, the frequency $f_{3\to1}$ of electromagnetic radiation emitted in the transition from $n = 3$ to $n = 1$ is

$$f_{3\to1} = \frac{E_3 - E_1}{h} = \frac{8 \times 10^{-32}\,\text{J}}{(6.63 \times 10^{-34}\,\text{J s})}$$

$$\approx 1.2 \times 10^2\,\text{Hz}$$

(remember from Unit 10 that $1\,\text{Hz}$ is $1\,\text{s}^{-1}$). The wavelength $\lambda_{3\to1}$ of the radiation emitted in the transition from $n = 3$ to $n = 1$ is given by the usual equation, $c = f\lambda$, where c is the speed of light in a vacuum, $3 \times 10^8\,\text{m s}^{-1}$:

$$\lambda_{3\to1} = \frac{3 \times 10^8\,\text{m s}^{-1}}{1.2 \times 10^2\,\text{Hz}}$$

$$\approx 2.5 \times 10^6\,\text{m}$$

ITQ 3 (a) $E = 12h^2/(8mL^2)$. Using Equation 17,

$$E = \frac{h^2}{8mL^2}(2^2 + 2^2 + 2^2) = \frac{12h^2}{8mL^2}$$

Note that this energy level is not degenerate: it corresponds only to the wavefunction characterized by $n_1 = 2$, $n_2 = 2$ and $n_3 = 2$. Check that you have shown this level correctly on Figure 15.

(b) Yes, because this energy level corresponds to more than one wavefunction. In fact the energy level corresponds to *three* wavefunctions (Figure 15).

ITQ 4 Because the model is a very simple representation of the hydrogen atom, it is unreasonable to expect the model's predictions of the energy levels of the electron to agree exactly with the corresponding data—it is reasonable to expect only approximate agreement. If the model is to be regarded as a useful representation of the hydrogen atom, it is reasonable to assert that the model's predictions should underestimate or overestimate the corresponding data on the electron's energy values by no more than, say, one or two orders of magnitude (i.e. one or two powers of ten). In other words, if the model is to be useful, the predictions should be between 10^{-2} times and 10^2 times the experimental values. This criterion is in our opinion reasonable—you are of course at liberty to disagree!

ITQ 5 The four lowest energy levels of the particle in the box are 4.5 eV, 9 eV, 13.5 eV and 16.5 eV. *Don't forget to mark these energy levels on Figure 19!*

Figure 15 shows that the four lowest energy levels of a particle of mass m in a cubic box whose sides are each of length L are $3h^2/(8mL^2)$, $6h^2/(8mL^2)$, $9h^2/(8mL^2)$ and $11h^2/(8mL^2)$. Because in the model $h^2/(8mL^2) = 1.5\,\text{eV}$ (Equation 22), it follows that the four lowest energy levels of the particle in the atom-sized box are $3 \times 1.5\,\text{eV} = 4.5\,\text{eV}$, $6 \times 1.5\,\text{eV} = 9\,\text{eV}$, $9 \times 1.5\,\text{eV} = 13.5\,\text{eV}$ and $11 \times 1.5\,\text{eV} = 16.5\,\text{eV}$.

ITQ 6 (a) See Table 5.

(b) The agreement between the predictions of the model and the data is fairly good—the agreement is well within two powers of ten. Hence, according to the subjective criterion used in the answer to ITQ 4, the model may reasonably be regarded as a useful representation of the hydrogen atom.

TABLE 5

Transition	Energy of emitted photon
E_3 to E_2	$E_3 - E_2 = 13.5\,\text{eV} - 9\,\text{eV} = 4.5\,\text{eV}$
E_4 to E_2	$E_4 - E_2 = 16.5\,\text{eV} - 9\,\text{eV} = 7.5\,\text{eV}$
E_5 to E_2	$E_5 - E_2 = 18\,\text{eV} - 9\,\text{eV} = 9\,\text{eV}$
E_6 to E_2	$E_6 - E_2 = 21\,\text{eV} - 9\,\text{eV} = 12\,\text{eV}$

ITQ 7 (a) The nucleus contains 30 protons, because its atomic number is 30.

(b) The total number of protons and neutrons that it contains is 68 and, from (a), it contains 30 protons. It therefore contains $68 - 30 = 38$ neutrons.

(c) $+4.8 \times 10^{-18}\,\mathrm{C}$. The nucleus contains 30 protons (positively charged) and 38 neutrons, which are not charged. Since the *magnitude* of the charge of the proton is the same as that of the electron, the total charge Q of the nucleus is:

$$\begin{aligned} Q &= +30 \times (1.602 \times 10^{-19}\,\mathrm{C}) \\ &\approx +4.8 \times 10^{-18}\,\mathrm{C} \end{aligned}$$

(d) 30. Because the atom is neutral and the magnitude of the charge of the proton is the same as that of the electron, the number of protons N_p in the nucleus must be the same as the number of electrons N_e that move around the nucleus. Since $N_\mathrm{p} = 30$ from (a), $N_\mathrm{e} = 30$ also.

(e) No. All isotopes of an element have the same atomic number. So because the atomic number of $^{68}_{30}\mathrm{Zn}$ (which is 30) differs from that of X (which is 31), X cannot be an isotope of zinc.

ITQ 8 (a) Two, because there are two points on Figure 25 that correspond to $Z = 17$. The locations of these points on the Figure tell us that the stable isotopes of chlorine contain 18 and 20 neutrons, respectively.

(b) Because there is no point corresponding to $Z = 43$, there are *no* completely stable isotopes of technetium!

ITQ 9 (a)

$$E = \frac{h^2}{8mL^2}(n_1{}^2 + n_2{}^2 + n_3{}^2) \qquad (17)^*$$

where n_1, n_2 and n_3 are each equal to any whole number, i.e. 1 or 2 or 3 ... etc.

(b) Protons and neutrons in nuclei have energy levels because they are *confined*. Equation 17 shows that the energy of a particle in a cubic box is quantized, i.e. the particle has energy levels. Hence, just as the model (in conjunction with Equation 17) was used to infer that hydrogen has energy levels because it contains a confined electron, so it is reasonable to use the model to assert that the nuclei have energy levels because they consist of confined particles (protons and neutrons).

ITQ 10 (a) $h^2/(8mL^2) = 3.29 \times 10^{-13}\,\mathrm{J}$.

The value of Planck's constant h is $6.63 \times 10^{-34}\,\mathrm{J\,s}$, and, according to the model, $m = 1.67 \times 10^{-27}\,\mathrm{kg}$, $L = 1.00 \times 10^{-14}\,\mathrm{m}$, so

$$\begin{aligned} \frac{h^2}{8mL^2} &= \frac{(6.63 \times 10^{-34}\,\mathrm{J\,s})^2}{8 \times (1.67 \times 10^{-27}\,\mathrm{kg}) \times (1.00 \times 10^{-14}\,\mathrm{m})^2} \\ &= 3.29 \times 10^{-13}\,\mathrm{J} \end{aligned}$$

(b) Because $1\,\mathrm{MeV} \approx 1.602 \times 10^{-13}\,\mathrm{J}$,

$$\frac{h^2}{8mL^2} = \frac{3.29 \times 10^{-13}\,\mathrm{J}}{1.602 \times 10^{-13}\,\mathrm{J\,MeV^{-1}}} \approx 2.1\,\mathrm{MeV}$$

Hence, you should have completed Equation 23 as

$$E = 2.1\,\mathrm{MeV} \times (n_1{}^2 + n_2{}^2 + n_3{}^2)$$

ITQ 11 (a) The four lowest energy levels are 6.3 MeV, 12.6 MeV, 18.9 MeV and 23.1 MeV. Figure 15 shows that the four lowest energy levels of a particle of mass m in a cubic box whose sides each have length L are $3h^2/(8mL^2)$, $6h^2/(8mL^2)$, $9h^2/(8mL^2)$ and $11h^2/(8mL^2)$. Because, in the nuclear particle-in-a-box model, $h^2/(8mL^2) = 2.1\,\mathrm{MeV}$, it follows that the four lowest energy levels of the particle of mass $1.67 \times 10^{-27}\,\mathrm{kg}$ in the nucleus-sized cubic box are $3 \times 2.1\,\mathrm{MeV} = 6.3\,\mathrm{MeV}$, $6 \times 2.1\,\mathrm{MeV} = 12.6\,\mathrm{MeV}$, $9 \times 2.1\,\mathrm{MeV} = 18.9\,\mathrm{MeV}$ and $11 \times 2.1\,\mathrm{MeV} = 23.1\,\mathrm{MeV}$. (This question is very similar to ITQ 5, which concerned a particle of mass $9.11 \times 10^{-31}\,\mathrm{kg}$ in an *atom*-sized cubic box.)

(b) The energy spacings agree quite well with the experimentally determined spacings of carbon's nuclear energy levels shown in Figure 27: the agreement is well within our criterion of two orders of magnitude (ITQ 4). Comparisons of the model's prediction with the lowest energy levels of other nuclei are also successful.

SAQ ANSWERS AND COMMENTS

SAQ 1 (a) Using Equation 17 with the values $h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$, $m = 1.25 \times 10^{-28}\,\mathrm{kg}$ and $L = 6.63 \times 10^{-6}\,\mathrm{m}$,

$$E = \frac{(6.63 \times 10^{-34}\,\mathrm{J\,s})^2}{8 \times (1.25 \times 10^{-28}\,\mathrm{kg}) \times (6.63 \times 10^{-6}\,\mathrm{m})^2} \times (n_1{}^2 + n_2{}^2 + n_3{}^2)$$

i.e. $E = (10^{-29}\,\mathrm{J}) \times (n_1{}^2 + n_2{}^2 + n_3{}^2)$

(b) The particle's four lowest energy levels are $3 \times 10^{-29}\,\mathrm{J}$, $6 \times 10^{-29}\,\mathrm{J}$, $9 \times 10^{-29}\,\mathrm{J}$ and $11 \times 10^{-29}\,\mathrm{J}$. According to Figure 15, the four lowest energy levels of a particle of mass m in a cubic box whose sides are each of length L are $3h^2/(8mL^2)$, $6h^2/(8mL^2)$, $9h^2/(8mL^2)$ and $11h^2/(8mL^2)$. Because $h^2/(8mL^2) = 10^{-29}\,\mathrm{J}$ (as you saw in part (a)) for the situation described in the question, it follows that the particle's four lowest energy levels are $3 \times 10^{-29}\,\mathrm{J}$, $6 \times 10^{-29}\,\mathrm{J}$, $9 \times 10^{-29}\,\mathrm{J}$ and $11 \times 10^{-29}\,\mathrm{J}$.

SAQ 2 (a) $5 \times 10^{-29}\,\mathrm{J}$. The energy of the photon is equal to the energy difference between the energy levels involved, i.e.

$$11 \times 10^{-29}\,\mathrm{J} - 6 \times 10^{-29}\,\mathrm{J} = 5 \times 10^{-29}\,\mathrm{J}.$$

(b) $7.5 \times 10^4\,\mathrm{Hz}$. The energy of a photon is given by the standard equation $E = hf$, so the frequency f the radiation emitted in the transition specified in the question is given by the energy of the photon divided by Planck's constant.

Using the answer to part (a),

$$f = \frac{5.00 \times 10^{-29}\,\text{J}}{6.63 \times 10^{-34}\,\text{J s}} \approx 7.5 \times 10^{4}\,\text{Hz}$$

(c) 4×10^3 m. The wavelength λ and frequency f of electromagnetic radiation are related by the standard equation $f\lambda = c$, where c is the speed of light in a vacuum, $3 \times 10^8\,\text{m s}^{-1}$, i.e.

$$\lambda = \frac{3.0 \times 10^{8}\,\text{m s}^{-1}}{7.5 \times 10^{4}\,\text{Hz}}$$

$$\approx 4 \times 10^{3}\,\text{m}$$

using the answer to part (b).

SAQ 3 (a) The model was formulated in order to derive insight into the energy levels of the electron in the hydrogen atom. A more complete description of the hydrogen atom was not used, simply because that would have been too difficult—the model was extremely convenient because the expression for the energy levels of a particle in a cubic box was known at the outset (Equation 17).

(b) No—we proceeded with the simplifying assumption that the hydrogen atom is cubic in the hope that the assumption would not render the model useless. The success of the model justified our use of the assumption—this is certainly *not* to say that the success of the model proved that hydrogen atoms are cubic! It is better to say that the model's success in accounting for the energy levels of the electron in the hydrogen atom implies that the details of the atom's shape are not crucial to an understanding of its energy levels.

(c) No—as we said in the answer to ITQ 4, it is reasonable to expect the crude model to give only approximate agreement with experiment.

SAQ 4 First, the electron in the hydrogen atom has energy levels because it is *confined*. Second, the differences between the electron's lowest energy levels are determined principally by the mass of the electron, by the size of the atom and by the value of Planck's constant.

SAQ 5 Statements (b) and (d) are correct. Statement (a) is false because *all* particles are subject to a gravitational interaction; statement (b) is correct—the strong interaction is the interaction that binds together protons and neutrons in nuclei; statement (c) is false because the magnitude of the strong interaction between two neutrons at a given separation is the same as that of the strong interaction between two protons (and that between a proton and a neutron) at the same separation; statement (d) is correct because the strong interaction has a range of only about 10^{-14} m, far less than 1 cm (10^{-2} m).

SAQ 6 (a) The strong interaction between protons is attractive, whereas the electrostatic interaction between them is repulsive (because they have electrical charges of the same sign).

(b) For nuclei that have an atomic number greater than 83, the *repulsive* electrostatic interaction between their constituent protons is sufficient to overcome the *attractive* strong interaction that binds the protons and neutrons together. Hence, these nuclei are not completely stable.

SAQ 7 (a) The average binding energy of the ${}^{10}_{4}\text{Be}$ nucleus per constituent proton and neutron is 6.498 MeV. This is the *total* binding energy of the ${}^{10}_{4}\text{Be}$ nucleus (64.98 MeV) divided by the mass number of the nucleus (10). Remember, the mass number gives the total number of protons and neutrons that the nucleus contains.

(b) The constituents of the ${}^{10}_{4}\text{Be}$ nucleus are less tightly bound than those of the ${}^{12}_{6}\text{C}$ nucleus because the former nucleus has the smaller binding energy per constituent proton and neutron (Figure 31).

SAQ 8 (a) Approximately 28.4 MeV. According to Figure 31, the binding energy of a ${}^{4}_{2}\text{He}$ nucleus per constituent proton and neutron is approximately 7.1 MeV. Because the ${}^{4}_{2}\text{He}$ nucleus contains a total of *four* protons and neutrons, the total energy required to unbind a ${}^{4}_{2}\text{He}$ nucleus is approximately (7.1 MeV) × 4 = 28.4 MeV.

(b) Approximately 28.4 MeV of energy are released—the amount of energy released when a nucleus is formed is equal to the minimum energy required to unbind the constituents of the nucleus.

(c) Approximately 5×10^{-29} kg. The amount of energy released when a ${}^{4}_{2}\text{He}$ nucleus is formed from its free constituents is approximately 28.4 MeV (part (b)), so Einstein's equation $E = mc^2$ implies that the mass that is equivalent to the energy released is

$$\begin{array}{l}\text{mass equivalent to}\\ \text{28.4 MeV of energy}\end{array} = \frac{28.4\,\text{MeV}}{c^2}$$

Because $1\,\text{MeV} \approx 1.60 \times 10^{-13}\,\text{J}$ and $c \approx 3.00 \times 10^{8}\,\text{m s}^{-1}$,

$$\begin{array}{l}\text{mass equivalent to}\\ \text{28.4 MeV of energy}\end{array} = \frac{28.4 \times (1.60 \times 10^{-13}\,\text{J})}{(3.00 \times 10^{8}\,\text{m s}^{-1})^2}$$

$$\approx 5 \times 10^{-29}\,\text{kg}$$

SAQ 9 (a) The equation that describes the decay is

$${}^{212}_{84}\text{Po} \longrightarrow {}^{208}_{82}\text{Pb} + {}^{4}_{2}\text{He}\ (\alpha\text{-particle})$$

(b) kinetic energy of nuclei formed

$$= \left(\begin{array}{c}\text{rest mass}\\ \text{of } {}^{212}_{84}\text{Po}\end{array} - \begin{array}{c}\text{rest mass}\\ \text{of } {}^{208}_{82}\text{Pb}\end{array} - \begin{array}{c}\text{rest mass}\\ \text{of } {}^{4}_{2}\text{He}\end{array}\right) \times c^2$$

where c is the speed of light in a vacuum. This equation is derived by considering the law of conservation of energy for the reaction in part (a). By analogy with Equation 28,

$$\begin{array}{l}\text{energy}\\ \text{equivalent to}\\ \text{rest mass of}\\ {}^{212}_{84}\text{Po}\end{array} = \begin{array}{l}\text{energy}\\ \text{equivalent to}\\ \text{rest mass of}\\ {}^{208}_{82}\text{Pb}\end{array} + \begin{array}{l}\text{energy}\\ \text{equivalent to}\\ \text{rest mass of}\\ {}^{4}_{2}\text{He}\end{array} + \begin{array}{l}\text{kinetic}\\ \text{energy of}\\ {}^{208}_{82}\text{Pb and}\\ {}^{4}_{2}\text{He}\end{array}$$

SAQ 10 β^+-decay; $^{37}_{20}\mathrm{Ca} \longrightarrow {}^{37}_{19}\mathrm{K} + {}^{0}_{1}\mathrm{e}^+ + \nu_e$

The potassium nucleus that is formed contains 19 protons and $37 - 19 = 18$ neutrons. The original isotope contained only 17 neutrons so, in this decay, a neutron must somehow have been created. This happens in β^+-decay when a proton in the original nucleus transforms into a neutron, with the emission of a positron and an electron neutrino. The mass number of the calcium isotope will be the same as that of the potassium isotope because the *total* number of protons and neutrons will be the same in both nuclei. Hence, the equation that describes the β^+-decay is

$$^{37}_{20}\mathrm{Ca} \longrightarrow {}^{37}_{19}\mathrm{K} + {}^{0}_{1}\mathrm{e}^+ + \nu_e$$

SAQ 11 In radioactive γ-decay, a photon (γ-ray) is emitted when a proton or neutron in a *nucleus* makes a transition from a higher energy level to a lower energy level, whereas when visible light is emitted from atoms, each photon is emitted when an electron in the *atom* makes a transition from one of its energy levels to another lower energy level. Because the energy differences of nuclear energy levels are normally much greater than those of atomic energy levels, a photon emitted in γ-decay normally has a much greater energy than a photon emitted in an electronic transition.

SAQ 12 Statement (c) is correct (see Figure 42).

Statement (a) is false: ionizing radiation is radiation that can ionize matter. The types of ionizing radiation include α-, β^--, and β^+-particles, none of which is a type of electromagnetic radiation. Statement (b) is false: the damaged DNA molecule *may* rejoin. Statement (d) is false: for example, if the cell is a gamete, a change in the cell's structure may lead to inherited disorders in descendants.

SAQ 13 (a) The difference between spontaneous fission and induced fission is that the former occurs spontaneously, as the name suggests, whereas the latter can occur only when energy is transferred to the nucleus concerned.

(b) $^{236}_{92}\mathrm{U} \longrightarrow {}^{56}_{26}\mathrm{Fe} + {}^{180}_{66}\mathrm{Dy}$

In order to answer this question, it is a good idea to begin by representing the reaction as

$$^{236}_{92}\mathrm{U} \longrightarrow {}^{A}_{26}\mathrm{Fe} + {}^{180}_{Z}\mathrm{Dy}$$

where the mass number of the formed nucleus of iron is denoted by A and the atomic number of the formed nucleus of dysprosium is denoted by Z. Because the total number of protons and neutrons and the total number of protons are the same before the reaction as they are afterwards, it is possible to calculate A and Z.

To find A: $236 = A + 180$, therefore $A = 56$

To find Z: $92 = 26 + Z$, therefore $Z = 66$

Hence, the equation of this reaction is:

$$^{236}_{92}\mathrm{U} \longrightarrow {}^{56}_{26}\mathrm{Fe} + {}^{180}_{66}\mathrm{Dy}$$

SAQ 14 (a) The equation that describes this fusion is

$$^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \longrightarrow 2{}^{1}_{1}\mathrm{H} + {}^{4}_{2}\mathrm{He}$$

(b) No, the total rest mass of the nuclei is not conserved. It is easy to see this by writing down the energy conservation equation for the reaction:

energy equivalent to the total rest mass of the two $^{3}_{2}$H nuclei	=	energy equivalent to the sum of the rest masses of the two $^{1}_{1}$H nuclei and the $^{4}_{2}$He nucleus	+	energy released in the reaction

Because energy is released in the reaction, the energy equivalent to the total rest mass before the reaction must be greater than the energy equivalent to the total rest mass after the reaction. Because Einstein's equation says that the energy equivalent to mass is directly proportional to mass ($E = mc^2$), it follows that the total rest mass before the reaction must itself be greater than the total rest mass after the reaction. (This result is generally true for any reaction in which energy is released.)

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Figure 39 Associated Press; *Figure 41(a)* Berkeley Nuclear Laboratories; *Figures 41(b) and 42* National Radiological Protection Board; *Figure 46* Argonne National Laboratory; *Figure 47* United Kingdom Atomic Energy Authority.

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